KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS
MATHEMATICAL DEPARTMENT

Math. 002 | Test#2 | Term(042)

Name: I.D.# Sec.#

SHOW ALL YOUR WORK

Q1. Find the center, vertices, foci, eccentricity and asymptotes for the hyperbola

$$4x^2 - 9y^2 - 16x + 54y - 29 = 0$$

$4x^2 - 16x - 9y^2 + 54y = 29$

$4[x^2 - 4x] - 9[y^2 - 6y] = 29$

$4[x^2 - 4x + 2^2 - 2^2] - 9[y^2 - 6y + 3^2 - 3^2] = 29$

$4[(x-2)^2 - 4] - 9[(y-3)^2 - 9] = 29$

$4(x-2)^2 - 16 - 9(y-3)^2 + 81 = 29$

$4(x-2)^2 - 9(y-3)^2 = 29 - 65$

$4(x-2)^2 - 9(y-3)^2 = -36$

$\frac{(y-3)^2}{4} - \frac{(x-2)^2}{9} = 1$ V

center $(2, 3)$

$a^2 = 4 \to \boxed{a = 2}$, $b^2 = 9 \to \boxed{b = 3}$

$c^2 = a^2 + b^2 = 13 \to c = \sqrt{13}$

vertices $(2, 3 \pm 2) \to (2, 1), (2, 5)$

foci $(2, 3 + \sqrt{13})$, $(2, 3 - \sqrt{13})$

$e = \frac{c}{a} = \frac{\sqrt{13}}{2}$

Asymptotes $y - 3 = \pm \frac{2}{3}(x - 2)$

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Q2. If (a,b) and (c,d) are a solution of the system of equations $\begin{cases} 14x^2 - 3xy - 2y^2 = 56 \\ 2x^2 - 5xy + 2y^2 = 56 \end{cases}$,

then find $a + b + c + d$

$$- \begin{array}{l} 14x^2 - 3xy - 2y^2 = 56 \quad \text{--- } 1 \\ 2x^2 - 5xy + 2y^2 = 56 \quad \text{--- } 2 \end{array}$$

$12x^2 + 2xy - 4y^2 = 0$

$6x^2 + xy - 2y^2 = 0$

$(3x + 2y)(2x - y) = 0$

$x = -\frac{2}{3}y$ or $x = \frac{1}{2}y$

Substitute in 2

$x = -\frac{2}{3}y \Rightarrow \frac{8}{9}y^2 + \frac{10}{3}y^2 + 2y^2 = 56$

$\frac{8 + 30 + 18}{9}y^2 = 56 \rightarrow \frac{56}{9}y^2 = 56$

$y^2 = 9 \rightarrow y = \pm 3$

$y = 3 \rightarrow x = -\frac{2}{3}(3) = -2$

$\boxed{(-2,3)}$

$y = -3 \rightarrow x = -\frac{2}{3}(-3) = 2$

$\boxed{(2,-3)}$

$x = \frac{1}{2}y \rightarrow \frac{2}{4}y^2 - \frac{5y^2}{2} + 2y^2 = 56$

$\frac{2 - 10 + 8}{4}y^2 = 56 \rightarrow 0 = 56$

reject

$-2 + 3 + 2 + (-3) = 0$

Q3 If $A = \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$, find A^3

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$A^3 = A \cdot A \cdot A = \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ -5 & 8 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 18 \\ -18 & 19 \end{bmatrix}$

Q4 Find the values of $\underline{a}$ if the system of the equations $\begin{cases} x - y + 2az = 1 \\ x + y = 1 \\ 2x + y + a^2 z = 2a \end{cases}$, 7

is dependent .

$$\left[\begin{array}{ccc|c} 1 & -1 & 2a & 1 \\ 1 & 1 & 0 & 1 \\ 2 & 1 & a^2 & 2a \end{array}\right] \xrightarrow[-2R_1+R_3]{-R_1-R_2} \left[\begin{array}{ccc|c} 1 & -1 & 2a & 1 \\ 0 & 2 & -2a & 0 \\ 0 & 3 & -4a+a^2 & 2a-2 \end{array}\right]$$

$$\tfrac{1}{2}R_2 \left[\begin{array}{ccc|c} 1 & -1 & 2a & 1 \\ 0 & 1 & -a & 0 \\ 0 & 3 & -4a+a^2 & 2a-2 \end{array}\right] \xrightarrow{-3R_2+R_3} \left[\begin{array}{ccc|c} 1 & -1 & 2a & 1 \\ 0 & 1 & -a & 0 \\ 0 & 0 & 3a-4a+a^2 & 2a-2 \end{array}\right]$$

$3a-4a+a^2=0$ and $2a-2=0$

$-a+a^2=0$ $\quad$ $a=1$

$a(-1+a)=0$ and $a=1$

$a=0$ or $a=1$ and $a=1$

$\therefore$ $a=1$

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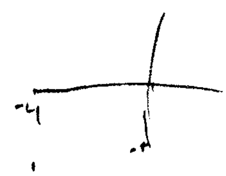
Q5. Find the standard form of the equation of the ellipse with foci at $(-4,-2)$ *and* $(0,-2)$ And passes through the point $P(-2,0)$

$2a = \sqrt{(-2+4)^2+(0+2)^2} + \sqrt{(-2-0)^2+(0+2)^2} = \sqrt{4+4} + \sqrt{4+4} = \sqrt{8}+\sqrt{8} = 2\sqrt{2}+2\sqrt{2}$

$2a = 4\sqrt{2} \rightarrow \boxed{a = 2\sqrt{2}}$

center $= \left(\frac{-4+0}{2}, \frac{-2+(-2)}{2}\right) = (-2,-2) \rightarrow \boxed{c = 2}$

$c^2 = a^2 - b^2 \rightarrow 4 = 4(2) - b^2 \rightarrow b^2 = 8-4 = 4 \rightarrow b^2 = 4$



H) $\frac{(x+2)^2}{8} + \frac{(y+2)^2}{4} = 1$

Q6. *If* $A = \begin{bmatrix} 1 & 2 & -3 & 0 \\ 4 & 2 & -3 & -2 \\ -1 & 0 & 2 & 1 \\ 3 & -4 & 0 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -3 \\ 2 & 4 \\ 0 & -5 \\ 1 & 2 \end{bmatrix}$, *and* $D = AB$, then $d_{12} - d_{41} =$ 5

$$d_{12} = \begin{bmatrix} 1 & 2 & -3 & 0 \end{bmatrix} \begin{bmatrix} -3 \\ 4 \\ -5 \\ 2 \end{bmatrix} \qquad d_{41} = \begin{bmatrix} 3 & -4 & 0 & -2 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 0 \\ 1 \end{bmatrix}$$

$= 6 - 8 + 0 - 2 = -4$

$= -3 + 8 + 15 + 0$

$= 20$

$\therefore d_{12} + d_{41} = 20 - (-4) = 24$

Q7. Solve the system of the equations $\begin{cases} 2x + 5y + 7z = -19 \\ 2x + 3y + 5z = -13 \\ x + y + 2z = -5 \end{cases}$ 10

$$\left[\begin{array}{ccc|c} 2 & 5 & 7 & -19 \\ 2 & 3 & 5 & -13 \\ 1 & 1 & 2 & -5 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & 1 & 2 & -5 \\ 2 & 3 & 5 & -13 \\ 2 & 5 & 7 & -19 \end{array}\right]$$

$$\xrightarrow[-2R_1+R_3]{-2R_1+R_2} \left[\begin{array}{ccc|c} 1 & 1 & 2 & -5 \\ 0 & 1 & 1 & -3 \\ 0 & 3 & 3 & -9 \end{array}\right] \xrightarrow{-3R_2+R_3} \left[\begin{array}{ccc|c} 1 & 1 & 2 & -5 \\ 0 & 1 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x + y + 2z = -5 \quad \cdots (1)$ infinitely many solution.

$y + z = -3 \quad \cdots (2)$

Let $z = c$, c is any real number

Substitute in (2) $\rightarrow y + c = -3 \rightarrow \boxed{y = -c - 3}$

Substitute in (1) $\rightarrow x + (-c - 3) + 2c = -5$

$x + c - 3 = -5 \rightarrow \boxed{x = -2 - c}$

solutions $(-2 - c, -c - 3, c)$.