Part I: (9 points) MULTIPLE CHOICE QUESTIONS: (MCQ)
[Bubble the correct answer on the OMR sheet]

1. For $f(x)=\left|3\sec\left(\frac{\pi x}{2}\right)\right|+1$, which one of the following statements is TRUE?

a) The range of *f* is $[4,\infty)$.

b) The period of *f* is 4.

c) The domain of *f* is $(-\infty,\infty)$.

d) The amplitude of *f* is 3.

2. Let n be any integer. The equation of the vertical asymptote of the function $f(x)=-2\csc\left(\frac{\pi x}{2}\right)$ is in the form:

a) $x=2n$

b) $x=2n+1$

c) $x=4n$

d) $x=(2n+1)\pi$

3. $\tan 105^\circ=$

a) $\frac{1+\sqrt{3}}{1-\sqrt{3}}$

b) $1+\sqrt{3}$

c) $2-\sqrt{3}$

d) $-2+\sqrt{3}$

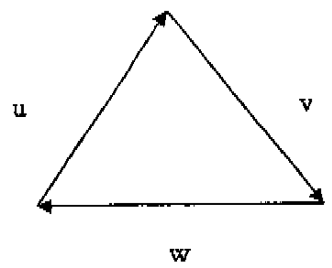
4. For the vectors u, v and w shown in the figure. Which one of the following relations is TRUE?

a) $u+v+w=0$

b) $v+w-u=0$

c) $w+u-v=0$

d) $u+v-w=0$



5. The set of solutions of the trigonometric equation $\sin 3x = 1$, where n is an integer, is:

a) $\frac{\pi}{6}(1+4n)$

b) $\frac{\pi}{3}(1+4n)$

c) $\frac{\pi}{6}(1+3n)$

d) $\frac{\pi}{6}(1+5n)$

6. Which one of the following is UNDEFINED?

a) $\cos\left(\cos^{-1}\frac{5}{3}\right)$

b) $\tan\left(\tan^{-1}\frac{5}{3}\right)$

c) $\cot\left(\cot^{-1}\frac{5}{3}\right)$

d) $\sec\left(\sec^{-1}\frac{5}{3}\right)$

<u>Part II:</u> (7 points) **[Fill in the blanks in the following questions]:**
[Show your steps]

1. Given $\cos 200^\circ = x$, then $\cos 100^\circ$ (in terms of x) is ....$-\frac{\sqrt{1+x}}{2}$

$\cos 100^\circ = \cos \frac{200^\circ}{2} = -\frac{\sqrt{1+\cos 200^\circ}}{2} = \frac{-\sqrt{1+x}}{2}$

2. For any integer n, $\cos((2n+1)\pi)=$..............-1....

$\cos(2n\pi + \pi) = \cos \pi = -1$

3. The <u>period</u> of the function $f(x)=\dfrac{1}{\cos 3x\cos x+\sin 3x\sin x}$ is ....π..

$\frac{1}{\cos(3x-x)} = \frac{1}{\cos 2x} = \sec 2x \Rightarrow \text{Period} = \frac{2\pi}{2} = \pi$

4. The <u>maximum value</u> of the function $f(x)=3\sin x-4\cos x$ is ....5.....

$\sqrt{3^2+4^2} = \sqrt{25} = 5$

5. If $\cos^{-1}x+\sin^{-1}\frac{\sqrt{2}}{2}=\frac{5\pi}{4}$, then $x=$ ....

$\cos^{-1}x + \frac{\pi}{4} = \frac{5\pi}{4} \Rightarrow \cos^{-1}x = \pi \Rightarrow x = \cos \pi = -1$

6. $\cos^{-1}\left(\cos\frac{7\pi}{5}\right)=$............$\frac{3\pi}{5}$......

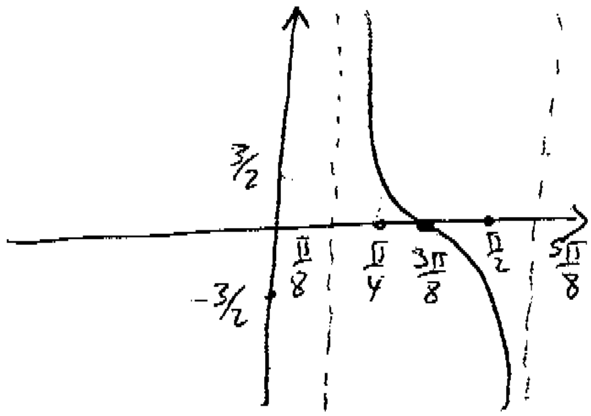
$\cos \frac{7\pi}{5} = \cos \frac{3\pi}{5} \Rightarrow \cos^{-1}\left(\cos \frac{7\pi}{5}\right) = \cos^{-1}\left(\cos \frac{3\pi}{5}\right) = \frac{3\pi}{5}$

7. If $u=\langle -1,1\rangle$ and $v=\langle 4,4\rangle$ are two vectors, then the smallest positive angle between u and v is .....$\frac{\pi}{2}$ or....90°

$u \cdot v = -4+4=0 \Rightarrow \theta = \cos^{-1} 0 = \frac{\pi}{2}$

Part III: WRITTEN QUESTIONS
[Provide neat and complete solution to each question. Show necessary steps for full credit.]

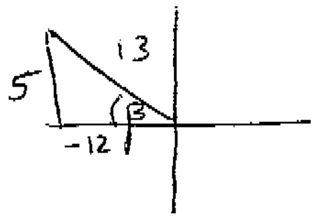
1. (5 points) Given the function $f(x) = \frac{3}{2}\cot\left(2x - \frac{\pi}{4}\right)$.
   a) The Period of f is: $= \frac{\pi}{2}$ (1 pt.)
   b) The Phase shift of f is: $= \frac{\frac{\pi}{4}}{2} = \frac{\pi}{8}$ (1 pt.)
   c) Use all the above to sketch the graph of f over one period.



(3 pts)

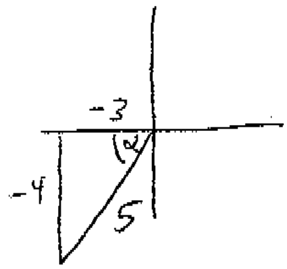
2. (4 points) Given $\cos\alpha = \frac{-3}{5}$, α in Quadrant III, and $\sin\beta = \frac{5}{13}$, β in Quadrant II, find $\sin\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$.

$\frac{1}{2}\, 2\sin\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) = \frac{1}{2}\sin(\alpha-\beta)$ (1 pt.)



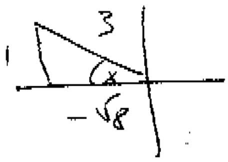
$= \frac{1}{2}\left[\sin\alpha\cos\beta - \cos\alpha\sin\beta\right]$ (1 pt.)

$= \frac{1}{2}\Big[\underbrace{\frac{-4}{5}\cdot\frac{-12}{13} - \frac{-3}{5}\cdot\frac{5}{13}}_{1\frac{1}{2}\text{ pts}}\Big] = \frac{1}{2}\left[\frac{48+15}{65}\right] = \frac{63}{130}$ ½ pt



3. (3 points) Given $\cos\left(\frac{\pi}{2}-x\right)=\frac{1}{3}$, where $\frac{3\pi}{4}\le x<\pi$, find the exact value of $\tan 2x$.

$\underbrace{\sin x=\frac{1}{3}}_{}$ 1 pt $\Rightarrow \tan x=\frac{1}{-\sqrt{8}}\cdot\frac{\sqrt{8}}{\sqrt{8}}=-\frac{\sqrt{8}}{8}$ ($\frac{1}{2}$ pt.)



$\tan 2x=\frac{2\tan x}{1-\tan^2 x}$ ($\frac{1}{2}$ pt.)

$=\frac{2\left(-\frac{\sqrt{8}}{8}\right)}{1-\frac{1}{8}}=\frac{-\frac{4\sqrt{2}}{8}}{\frac{7}{8}}=-\frac{4\sqrt{2}}{7}$ (1 pt.)

4. (4 points) Find the solution set of the trigonometric equation $\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)=1-\cos\left(x+\frac{\pi}{3}\right)$, where $0\le x<2\pi$. (Hint: you can use the half-angle Identities)

$\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)=\tan\frac{1}{2}\left(x+\frac{\pi}{3}\right)=\frac{\sin\left(x+\frac{\pi}{3}\right)}{1+\cos\left(x+\frac{\pi}{3}\right)}=1-\cos\left(x+\frac{\pi}{3}\right)$ (1 pt.)

$\sin\left(x+\frac{\pi}{3}\right)=1-\cos^2\left(x+\frac{\pi}{3}\right)=\sin^2\left(x+\frac{\pi}{3}\right)$

$\sin\left(x+\frac{\pi}{3}\right)\left(\sin\left(x+\frac{\pi}{3}\right)-1\right)=0.$ (1 pt.)

$\sin\left(x+\frac{\pi}{3}\right)=0 \Rightarrow x+\frac{\pi}{3}=n\pi \Rightarrow x=n\pi-\frac{\pi}{3}$

$x=\frac{2\pi}{3}$ X or $x=\frac{5\pi}{3}$ ✓ ($\frac{1}{2}$ pt.)

$\sin\left(x+\frac{\pi}{3}\right)=1 \Rightarrow x+\frac{\pi}{3}=\frac{\pi}{2}+n\pi \Rightarrow x=n\pi+\frac{\pi}{6}$

$x=\frac{\pi}{6}$ ✓ or $x=\frac{7\pi}{6}$ X ($\frac{1}{2}$ pt.)

Check $\Rightarrow$ $S.S=\left\{\frac{\pi}{6},\frac{5\pi}{3}\right\}$ (1 pt.)

5. (3 points) Given the vectors $u = 3i + 4j$ and $v = 2i + j$, find

a) The dot product $u \cdot v$

$$u \cdot v = 6 + 4 = 10 \quad (1\text{pt.})$$

b) The magnitudes of u and v.

$$\left.\begin{aligned} \|u\| &= \sqrt{9+16} = 5 \\ \|v\| &= \sqrt{4+1} = \sqrt{5} \end{aligned}\right\} (1\text{pt.})$$

c) The value of $proj_v u$.

$$proj_v u = \frac{u \cdot v}{\|v\|} = \frac{10}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{10\sqrt{5}}{5} = 2\sqrt{5} \quad (1\text{pt.})$$

6. (3 points) Verify the identity $\dfrac{\cos^2 x - \cos 2x}{1 + \cos x} = 2\sin^2 \dfrac{x}{2}$.

(1pt.)

$$LHS \quad \frac{\cos^2 x - (2\cos^2 x - 1)}{1 + \cos x} = \frac{1 - \cos^2 x}{1 + \cos x} = \frac{(1 - \cos x)(1 + \cos x)}{1 + \cos x}$$

$$= 1 - \cos x = 2\left(\frac{1 - \cos x}{2}\right) = 2\sin^2 \frac{x}{2}.$$

(1pt.) (1pt.)

7. (3 points) If $\sin 40^\circ + \cos 40^\circ = k\sin(\beta)$, then find the values of k and β.

$\sin 40^\circ + \cos 40^\circ = \sqrt{2}\,\sin(40^\circ + \alpha)$ where (1pt.)

$\sin\alpha = \frac{1}{\sqrt{2}}$ & $\cos\alpha = \frac{1}{\sqrt{2}} \Rightarrow \alpha = 45^\circ$ (1pt.)

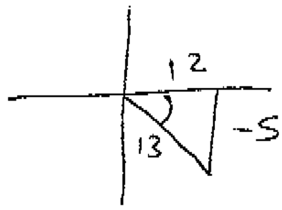
$\therefore \sin 40^\circ + \cos 40^\circ = \sqrt{2}\,\sin 85^\circ$

$k = \sqrt{2}$ & $\beta = 85^\circ$ (1pt.)

8. (3 points) Find the value of $\cot\left(2\csc^{-1}\frac{-13}{5}\right)$.

$$\cot\left(2\csc^{-1}\frac{-13}{5}\right) = \frac{1}{\tan\left(2\csc^{-1}\frac{-13}{5}\right)}$$

$$= \frac{1-\tan^2\left(\csc^{-1}\frac{-13}{5}\right)}{2\tan\left(\csc^{-1}\frac{-13}{5}\right)} \quad \text{(1pt.)}$$



$$= \frac{1-\left[\frac{-5}{12}\right]^2}{2\left(\frac{-5}{12}\right)} \text{ (1pt.)} = \frac{1-\frac{25}{144}}{\frac{-10}{12}}$$

$$= \frac{119}{144}\cdot\frac{12}{-10} = \frac{-119}{120} \quad \text{(1pt.)}$$