King Fahd University of Petroleum and Minerals
College of Sciences
Prep-Year Math Program

Code: Master

Math 002 Exam II
Term 022 (2002-2003)
Sunday, April 27, 2003
Time Allowed: 90 Minutes

Code: Master

Student's Name: ....

ID #: .... **Section #:** ....

This exam consists of two parts:

Part I : Multiple Choice Bubble the correct answer on the OMR sheet

Part II : Written Provide neat and complete solutions. Show all necessary steps for full credit.

Calculators, pagers, or mobiles are NOT allowed during this examination.

Question	Points	Student's Score	Grader
Part I: Multiple Choice	12		
Part II: Written			
1	8		Mr Al-Absi
2	4		Mr Ahmad
3	3		Dr Al-Attas
4	3		Mr Al-Humaidi
5	3		Mr Maslamani
6	3		Mr Saifullah
7	4		Mr Shehadeh
8	4		Mr Yushau

Total	
	44

MASTER

MATH 002 – T022 (EXAM II)

Part I: (12-pionts) MULTIPLE CHOICE QUESTIONS (MCQ)

[Bubble the correct answer on the OMR sheet]

1. For $0 \le x \le 4\pi$, the graph of $y = -3\sec\frac{x}{2}$ lies completely below the x-axis on the interval:

$a)$ $[0,\pi) \cup (3\pi, 4\pi]$
$b)$ $(\pi, 3\pi)$
$c)$ $[0,\pi) \cup (\frac{3\pi}{2}, 2\pi]$
$d)$ $(\frac{\pi}{2}, \frac{3\pi}{2})$

2. The expression $\sqrt{3}\sin 10^0 + \cos 10^0$ is equal to:

$a)$ $2\sin 40^0$
$b)$ $2\sin 70^0$
$c)$ $2\sin 100^0$
$d)$ $4\sin 20^0$

3. The expression $\dfrac{\sin\theta}{\csc\theta - \cot\theta}$ is identical to:

$a)$ $1+\cos\theta$
$b)$ $1-\cos\theta$
$c)$ $1+\sin\theta$
$d)$ $1-\sin\theta$

4. The domain D and the range R of the function $f(x) = -3\sin^{-1}(2x-1)$ are:

a) $D = [0,1]$; $R = [-\frac{3\pi}{2}, \frac{3\pi}{2}]$

b) $D = [-1,1]$; $R = [-\frac{\pi}{2}, \frac{\pi}{2}]$

c) $D = [-1,1]$; $R = [-\pi, \pi]$

d) $D = [0,1]$; $R = [-\frac{\pi}{6}, \frac{\pi}{6}]$

5. The expression: $\sin 13^0 \sin 73^0 + \sin 77^0 \sin 17^0$ is equal to:

a) $\frac{1}{2}$

b) $\frac{\sqrt{3}}{2}$

c) $-\frac{1}{2}$

d) $-\frac{\sqrt{3}}{2}$

6. The number of the vertical asymptotes to the graph of $y = \tan(\frac{\pi}{2} + 2x)$ over the interval $[0, \pi]$ is equal to:

a) 3

b) 2

c) 1

d) 4

7. Which one of the following statements is FALSE:

a) $\sin^{-1}(\sin\frac{4\pi}{5}) = \frac{4\pi}{5}$

b) $\tan^{-1}(\tan\frac{\pi}{5}) = \frac{\pi}{5}$

c) $\cos^{-1}(\cos\frac{4\pi}{5}) = \frac{4\pi}{5}$

d) $\tan^{-1}(\tan\frac{4\pi}{5}) = -\frac{\pi}{5}$

8. The number of solutions of the equation $\sin^2 x = \frac{1}{2}\sin 2x$ over the interval $[0, \frac{3\pi}{2})$ is equal to:

a) 4
b) 3
c) 2
d) 1

MATH 002 – T022 (EXAM II)

PART II: WRITTEN QUESTIONS

Provide neat and complete solution to each question. Show necessary steps for full credit.

1. (8-points) Given the function

$$f(x) = -2\cos(\frac{\pi}{2}x - \frac{\pi}{2}) + 2$$

a) The period of f is: $= 2\pi/|b| = 2\pi/(\pi/2) = 4$ --- 1 Point

b) The phase shift of the graph of f is: $= -c/b = -(-\pi/2)/(\pi/2) = 1$ --- 1 point

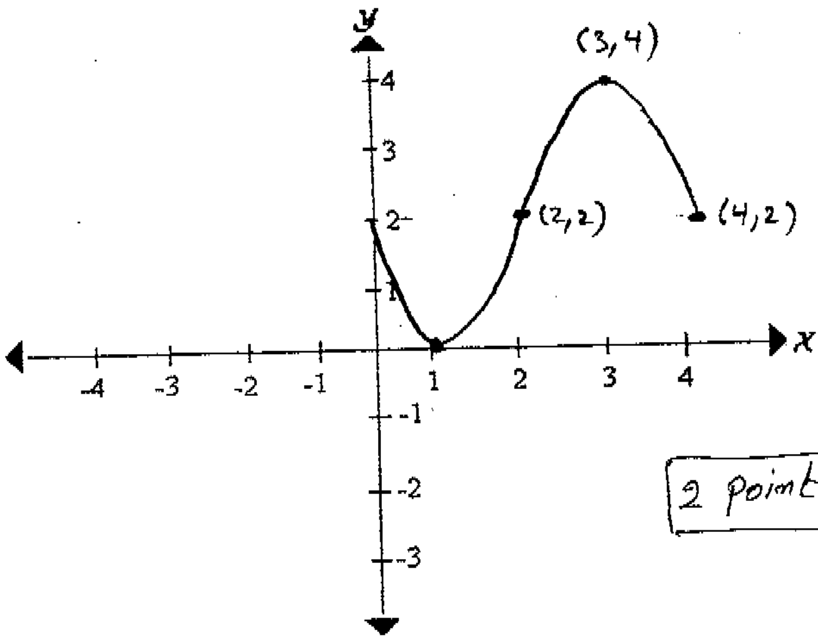
c) The vertical translation of the graph of f is: $=$ 2 units up --- 1 Point

d) The amplitude of the graph of f is: $= |a| = |-2| = 2$ --- 1 Point

e) The domain of f is: $= (-\infty, \infty)$ --- 1 Point

f) The range of f is: $= [0, 4]$ --- 1 point

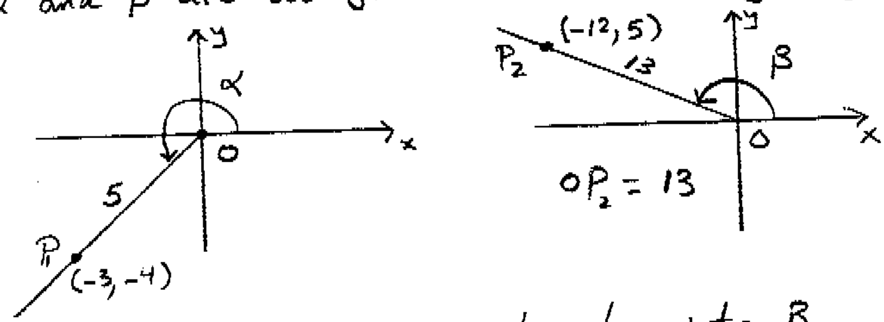
g) Use all the above to sketch the graph of f over the interval [0,4].



$\boxed{\text{2 points}}$

2. (4-points) Given $\sin\alpha = -\frac{4}{5}$, α in Quadrant III, and $\cos\beta = -\frac{12}{13}$, β in Quadrant II.
Find the exact value of $\cot(\alpha+\beta)$

α and β are as given in the following figures :



$OP_1 = 5$ $OP_2 = 13$ --- 2 points

$$\Rightarrow \cot(\alpha+\beta) = \frac{1}{\tan(\alpha+\beta)} = \frac{1-\tan\alpha\tan\beta}{\tan\alpha + \tan\beta} \quad \text{--- 1 point}$$

$$= \frac{1-\left(\frac{4}{3}\right)\left(-\frac{5}{12}\right)}{\frac{4}{3}+\left(-\frac{5}{12}\right)}$$

$$= \frac{36+20}{48-15} = \frac{56}{33} \quad \text{--- 1 point}$$

3. (3-points) Verify the identity $\frac{\cos 2x + \cos x}{\sin 2x - \sin x} = \csc x + \cot x$

$$LHS = \frac{\cos 2x + \cos x}{\sin 2x - \sin x} = \frac{2\cos^2 x - 1 + \cos x}{2\sin x \cos x - \sin x} \quad \text{--- 1 Point}$$

$$= \frac{(2\cos x - 1)(1+\cos x)}{\sin x(2\cos x - 1)} = \frac{1+\cos x}{\sin x} \quad \text{--- 1 Point}$$

$$= \frac{1}{\sin x} + \frac{\cos x}{\sin x} = \csc x + \cot x = RHS$$

--- 1 Point

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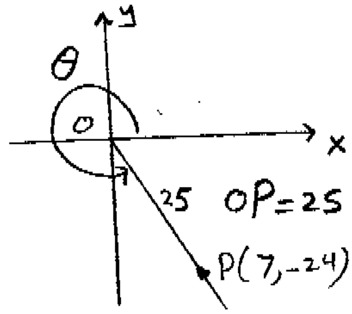
4. (3-points) If $\csc\theta = -\frac{25}{24}$, $\frac{7\pi}{2} < \theta < 4\pi$, find the exact value of $\sin\frac{\theta}{2}$

$\frac{7\pi}{2} < \theta < 4\pi \Rightarrow \frac{7\pi}{4} < \frac{\theta}{2} < 2\pi \Rightarrow \theta$ and $\theta/2$ are both in Quadrant IV and θ as in the figure ... 1.5 Points

$$\Rightarrow \sin\frac{\theta}{2} = -\sqrt{\frac{1-\cos\theta}{2}} = -\sqrt{\frac{1-\frac{7}{25}}{2}} = -\sqrt{\frac{18}{50}} = -\sqrt{\frac{9}{25}} = -\frac{3}{5}$$ ... 1.5 points



5. (3-points) Solve the inverse trigonometric equation: $\cos^{-1}x + \tan^{-1}\frac{5}{12} - \frac{\pi}{2} = 0$

$\Rightarrow \cos^{-1}x = \frac{\pi}{2} - \tan^{-1}\frac{5}{12}$

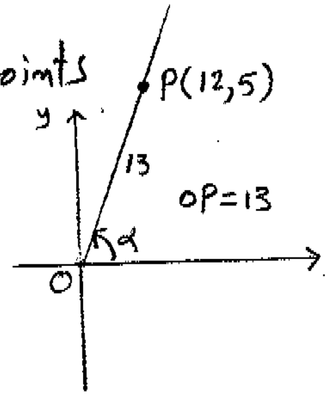
$\Rightarrow \cos(\cos^{-1}x) = \cos\left(\frac{\pi}{2} - \tan^{-1}\frac{5}{12}\right)$

$\Rightarrow x = \sin\left(\tan^{-1}\frac{5}{12}\right)$. ... 1.5 points

Let $\alpha = \tan^{-1}\frac{5}{12}$

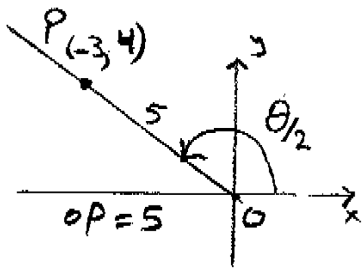
$\Rightarrow \alpha$ as in the figure

$\Rightarrow x = \sin\alpha = \frac{5}{13}$ ... 1.5 points



6. (3-points) If $\cos\frac{\theta}{2} = -\frac{3}{5}$, $\pi < \theta < \frac{3\pi}{2}$, find the exact value of $\tan\theta$

$\pi < \theta < \frac{3\pi}{2} \Rightarrow \frac{\pi}{2} < \frac{\theta}{2} < \frac{3\pi}{4} \Rightarrow$ $\frac{\theta}{2}$ is in Quadrant II and as given in the figure ... 1.5 points



$$\Rightarrow \tan\theta = \frac{2\tan\theta/2}{1-\tan^2\theta/2} = \frac{2(-4/3)}{1-(-4/3)^2} = \frac{-24}{9-16} = \frac{24}{7}$$ 1.5 points

$\boxed{OR}$ $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{2\sin\theta/2\cos\theta/2}{2\cos^2\theta/2 - 1}$ ...etc.

7. (4-points) Find the exact solutions of the equation $\tan\frac{x}{2} = \sin x, \quad 0 \le x < 2\pi$

$$\tan\frac{x}{2} = \sin x \Rightarrow \frac{1-\cos x}{\sin x} = \sin x$$ --- 1 point

$$\left.\begin{aligned} &\Rightarrow 1-\cos x = \sin^2 x \\ &\Rightarrow 1-\cos x = 1-\cos^2 x \\ &\Rightarrow \cos x = \cos^2 x \end{aligned}\right\}$$ --- 1 point

$$\left.\begin{aligned} &\Rightarrow \cos x(\cos x - 1) = 0 \\ &\Rightarrow \cos x = 0 \quad \text{or} \quad \cos x = 1 \end{aligned}\right\}$$ --- 1 point

$$\left.\begin{aligned} &\Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2} \quad \text{or} \quad x = 0 \\ &\Rightarrow \text{The solutions are: } 0, \frac{\pi}{2}, \frac{3\pi}{2} \end{aligned}\right\}$$ --- 1 point

Important Note: Some students may use $\frac{\sin x}{1+\cos x} = \sin x$

$\Rightarrow$ Possible solutions are $0, \pi, \pi/2, 3\pi/2$

$\Rightarrow$ π must be rejected since $\tan\frac{\pi}{2}$ is undefined.

8. (4-points) Find the exact value of the expression: $\left(\cos\frac{4\pi}{9} - \cos\frac{\pi}{9}\right)^2 + \left(\sin\frac{4\pi}{9} - \sin\frac{\pi}{9}\right)^2$

$$\left.\begin{aligned} \text{The expression} = {} & \cos^2\frac{4\pi}{9} - 2\cos\frac{4\pi}{9}\cos\frac{\pi}{9} + \cos^2\frac{\pi}{9} \\ & + \sin^2\frac{4\pi}{9} - 2\sin\frac{4\pi}{9}\sin\frac{\pi}{9} + \sin^2\frac{\pi}{9} \end{aligned}\right\}$$ ... 1 point

$$\left.\begin{aligned} &= \left(\cos^2\frac{4\pi}{9} + \sin^2\frac{4\pi}{9}\right) + \left(\cos^2\frac{\pi}{9} + \sin^2\frac{\pi}{9}\right) \\ &\quad - 2\left(\cos\frac{4\pi}{9}\cos\frac{\pi}{9} + \sin\frac{4\pi}{9}\sin\frac{\pi}{9}\right) \end{aligned}\right\}$$ ... 1 point

$$\left.\begin{aligned} &= 1 + 1 - 2\cos\left(\frac{4\pi}{9} - \frac{\pi}{9}\right) \\ &= 2 - 2\cos\frac{\pi}{3} \\ &= 2 - 2\left(\tfrac{1}{2}\right) = 1 \end{aligned}\right\}$$ ... 2 points