

Q<sub>1</sub>

$$f(x) = \left(\frac{1}{3}\right)^{x-2} - 1$$

x	0	2	3
y	8	0	$-\frac{2}{3}$

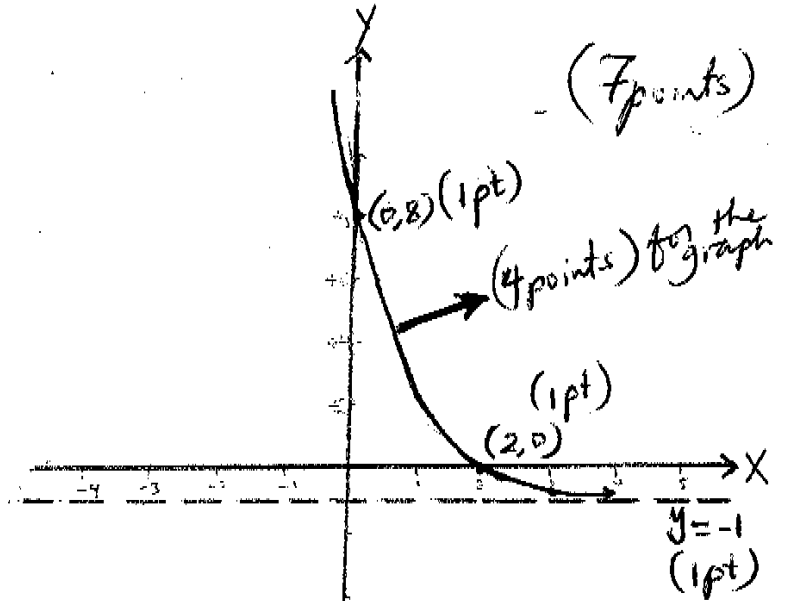
OR

Graph  $f(x) = \left(\frac{1}{3}\right)^x$   
as decreasing exp.  
then

- a) shift right 2 units  
b) shift down 1 unit

x-int  $\Rightarrow$  let  $y=0 \Rightarrow \left(\frac{1}{3}\right)^{x-2} = 1 \Rightarrow \therefore \boxed{x-2=0} \Rightarrow \boxed{x=2} \Rightarrow (2,0)$

y-int  $\Rightarrow$  let  $x=0 \Rightarrow y = \left(\frac{1}{3}\right)^{-2} - 1 = 9 - 1 = 8 \Rightarrow (0,8)$



Q<sub>2</sub> IF  $\log_2 5 = X$  and  $\log_2 3 = Y$ , find  $\log_{\sqrt{2}} 300$  (6 points)

$$\begin{aligned} \log_{\sqrt{2}} 300 &= \frac{\log_2 300}{\log_2 \sqrt{2}} = \frac{\log_2 300}{\log_2 2^{\frac{1}{2}}} = 2 \log_2 300 \quad (2 \text{ pts}) \\ &= 2 [\log_2 3 + \log_2 100] \quad (2 \text{ pts}) \\ &= 2 [\log_2 3 + \log_2 10^2] \\ &= 2 [\log_2 3 + 2 \log_2 10] \quad (1 \text{ pt}) \\ &= 2 [\log_2 3 + 2 \log_2 5 + 2 \log_2 2] \\ &= 2 [Y + 2X + 2] \\ &= 4X + 2Y + 4 \quad (1 \text{ pt}) \end{aligned}$$

Q3

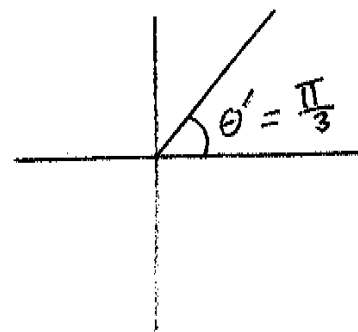
$$\begin{aligned} \frac{\sin t}{1 + \cos t} + \frac{1 + \cos t}{\sin t} &= \frac{\sin^2 t + (1 + \cos t)^2}{(\sin t)(1 + \cos t)} \quad (2 \text{ pts}) \quad (6 \text{ points}) \\ &= \frac{\sin^2 t + 1 + 2 \cos t + \cos^2 t}{(\sin t)(1 + \cos t)} \quad (1 \text{ pt.}) \\ &= \frac{2 + 2 \cos t}{(\sin t)(1 + \cos t)} \quad (1 \text{ pt.}) \\ &= \frac{2(1 + \cos t)}{(\sin t)(1 + \cos t)} = \frac{2}{\sin t} = 2 \csc t \end{aligned}$$

↓ (2 pts)

Q4 Find the values of  $x$  and  $y$  such that  $w(-\frac{7\pi}{3}) = (x, y)$ 

$$x = \cos\left(-\frac{7\pi}{3}\right) = \cos\left(\frac{7\pi}{3}\right) = \cos\frac{\pi}{3} = \frac{1}{2} \quad (3 \text{ pts}) \quad (6 \text{ pts})$$

$$y = \sin\left(-\frac{7\pi}{3}\right) = -\sin\left(\frac{7\pi}{3}\right) = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2} \quad (3 \text{ pts})$$



$$Q_5 \quad \log_3(-x) + \log_3(6-x) = 3$$

(6 points)

$$\Rightarrow \log_3 [(-x)(6-x)] = 3 \quad (2 \text{ pts})$$

$$\Rightarrow (-x)(6-x) = 3^3 \quad (1 \text{ pt.})$$

$$\Rightarrow x^2 - 6x - 27 = 0$$

$$\Rightarrow (x-9)(x+3) = 0 \quad (1 \text{ pt.})$$

$$\Rightarrow x-9 = 0 \quad \text{or} \quad x+3 = 0$$

$$\Rightarrow x = 9 \quad \text{or} \quad x = -3$$

check! will give the solution set =  $\{-3\}$   
(1 pt.) (1 pt.)

Q6 a) Find the exact value of  $\sin^2 45^\circ + \cos^2 60^\circ$  (3 points)

$$\begin{aligned} \sin^2 45^\circ + \cos^2 60^\circ &= \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{1}{2} + \frac{1}{4} \\ &\quad (1 \text{ pt.}) \quad (1 \text{ pt.}) \\ &= \frac{3}{4} \quad (1 \text{ pt.}) \end{aligned}$$

b) A wheel is rotating at 100 revolutions per minute. Find the angular speed in radian per second. (5 points)

$$\begin{aligned} 100 \text{ rev/minute} &= \left(\frac{100 \text{ rev}}{1 \text{ minute}}\right) \left(\frac{2\pi \text{ radians}}{1 \text{ rev}}\right) \left(\frac{1 \text{ minute}}{60 \text{ seconds}}\right) \\ &\quad (2 \text{ pts}) \quad (1 \text{ pt.}) \\ &= \frac{10\pi}{3} \text{ rad/s} \\ &\quad (1 \text{ pt.}) \quad (1 \text{ pt.}) \end{aligned}$$

Q7  $P(t) = 16000 e^{2t}$ . Find the time when the population reaches 48000. (5 points)

$$48000 = 16000 e^{2t} \Rightarrow e^{2t} = 3 \Rightarrow 2t = \ln 3 \quad (\text{pt.})$$

(2 pts)

$$\Rightarrow t = \frac{1}{2} \ln 3$$

$$\Rightarrow t = \ln \sqrt{3} \quad \text{Years.}$$

(1 pt.) (1 pt.)

Q8  $f(x) = -\frac{1}{2} + \log_q(1-2x)$  (9 points)

a) Domain:  $1-2x > 0$   
 $-2x > -1$

$$x < \frac{1}{2} \quad \text{or} \quad D = (-\infty, \frac{1}{2})$$

b) Range:  $R = (-\infty, \infty)$

c) Vertical Asymptote:  $1-2x=0 \Rightarrow x = \frac{1}{2}$

d) X-int  $\Rightarrow y=0 \Rightarrow -\frac{1}{2} + \log_q(1-2x) = 0$

$$\Rightarrow \log_q(1-2x) = \frac{1}{2}$$

$$\Rightarrow 1-2x = 3 \Rightarrow -2x = 2 \Rightarrow x = -1$$

$\Rightarrow (-1, 0)$

e) Y-int  $\Rightarrow x=0 \Rightarrow y = -\frac{1}{2} + \log_q 1 = -\frac{1}{2}$

$$\Rightarrow (0, -\frac{1}{2})$$

Q.8 (a), (b), (c)  $f(x) = -\frac{1}{2} + \log_9(1-2x)$  ((5 points))

a) Domain,  $1-2x > 0$  (1pt)  
 $-2x > -1$   
 $x < \frac{1}{2} \Rightarrow D: (-\infty, \frac{1}{2})$  (1pt.)

b) Range:  $\mathbb{R}$  or  $(-\infty, \infty)$  or All real numbers ((1 point))

c) Vertical Asymptote:  $1-2x=0 \Rightarrow \boxed{x=\frac{1}{2}}$  (1pt.)

Q.8 (d), (e)  $f(x) = -\frac{1}{2} + \log_9(1-2x)$  ((4 points))

d) X-int, let  $y=0 \Rightarrow -\frac{1}{2} + \log_9(1-2x) = 0$   
 $\Rightarrow \log_9(1-2x) = \frac{1}{2} \Rightarrow$  (2pts)  
 $1-2x = 9^{\frac{1}{2}}$   
 $1-2x = 3$   
 $x = -1$   
 $\Rightarrow$  X-int  $(-1, 0)$

e) y-int, let  $x=0 \Rightarrow y = -\frac{1}{2} + \log_9 1 = -\frac{1}{2} + 0$  (2pts)  
 $= -\frac{1}{2}$   
 $\Rightarrow$  y-int  $(0, -\frac{1}{2})$

Q9

$f(x) = a \cos bx$ ,  $b > 0$ , Period = 6 and  $f(3) = 4$   
Find  $f(\frac{21}{4})$ . (7 points)

$$\text{Period} = \frac{2\pi}{b} = 6 \Rightarrow b = \frac{2\pi}{6} = \frac{\pi}{3} \quad (2 \text{ pts})$$

$$\text{also, } f(3) = a \cos\left(\frac{\pi}{3}\right)(3) = a \cos \pi = -a = 4$$

$$\therefore a = -4 \quad (2 \text{ pts})$$

$$\text{So, } f(x) = -4 \cos\left(\frac{\pi}{3}x\right) \quad (1 \text{ pt.})$$

$$\begin{aligned} \text{and } f\left(\frac{21}{4}\right) &= -4 \cos\left(\frac{\pi}{3} \cdot \frac{21}{4}\right) = -4 \cos \frac{7\pi}{4} \\ &= -4 \cos \frac{\pi}{4} \\ &= (-4)\left(\frac{\sqrt{2}}{2}\right) = -2\sqrt{2} \quad (2 \text{ pts}) \end{aligned}$$

Q10

$$\text{a) } d = 14 \text{ cm} \Rightarrow r = 7 \text{ cm} \quad (1 \text{ pt}) \quad (5 \text{ points})$$

$$\theta = 150^\circ = (150^\circ) \left(\frac{\pi \text{ radians}}{180^\circ}\right) = \frac{5\pi}{6} \text{ radians} \quad (1 \text{ pt.})$$

$$\underbrace{S = r\theta}_{(1 \text{ pt.})} = (7) \left(\frac{5\pi}{6}\right) = \frac{35\pi}{6} \text{ cm.} \quad \begin{matrix} (6) \\ (1 \text{ pt.}) \end{matrix} \quad \begin{matrix} (1 \text{ pt.}) \end{matrix}$$

$$\text{b) Find } x \text{ if } 5 \log_2 (\log_4 16) + x = 1 + 2 \ln e^x \quad (5 \text{ points})$$

$$5 \log_2 (\log_4 4^2) + x = 1 + 2x \quad (3 \text{ pts})$$

$$5 \log_2 2 + x = 1 + 2x$$

$$5 + x = 1 + 2x \quad (1 \text{ pt.})$$

$$\boxed{x = 4} \quad (1 \text{ pt.})$$

Q11

$$a) f(x) = x - \sin x \quad (4 \text{ points})$$

$$f(-x) = -x - \sin(-x) = -x + \sin x \quad (2 \text{ pts})$$

$$= -(x - \sin x) = -f(x) \quad \therefore f \text{ is an odd function} \quad (1 \text{ pt})$$

b)  $6\frac{1}{3}$  terminates in the First quadrant  $(4 \text{ points})$

$$2\pi \approx 6.28 < 6\frac{1}{3} < 7.85 \approx \frac{5\pi}{2}$$

Q12

$$y = -\left|3 \sin \frac{2x}{3}\right| \quad (4 \text{ points})$$

Graph first  $y = 3 \sin \frac{2x}{3}$

$$\text{Amp} = |3| = 3$$

$$\text{period} = \frac{2\pi}{|b|} = \frac{2\pi}{\frac{2}{3}} = 3\pi$$

For  $y = -\left|3 \sin \frac{2x}{3}\right|$

- values of  $y$  will be  $\ominus$
- period =  $\frac{3\pi}{2}$

Period	max	min
$\frac{3\pi}{2}$	0	-3

(1pt) (1pt) (1pt)

