King Fahd University of Petroleum and Minerals
Math Prep-Year Program Math 002

Term 051 **Quiz# 3** **Sec.03**

Solutions

Q1. Given $f(x) = -3\sec\left(\frac{2x}{3} - \frac{\pi}{3}\right) - 2$, then

a) The **range** of the function f is

$$(-\infty, -3-2] \cup [3-2, \infty) = (-\infty, -5] \cup [1, \infty)$$

b) The **phase shift** of f is

$$-\frac{c}{b} = -\frac{-\pi/2}{2/3} = \frac{3\pi}{4}$$

c) The **period** of f is

$$\frac{2\pi}{2/3} = 3\pi$$

Q2. Given $g(x) = 3\tan\left(\pi x - \frac{\pi}{3}\right) - 5$, find the equation **of vertical asymptotes** of g on the interval $[-2, 3]$

$$g(x) = \frac{3\sin\left(\pi x - \frac{\pi}{3}\right)}{\cos\left(\pi x - \frac{\pi}{3}\right)} - 5$$

$$\cos\left(\pi x - \frac{\pi}{3}\right) = 0 \rightarrow \left(\pi x - \frac{\pi}{3}\right) = \frac{\pi}{2} \rightarrow \pi x = \frac{5\pi}{6} \rightarrow x = \frac{5}{6}$$

$$V.A. \rightarrow x = \frac{5}{6} + n(period) = \frac{5}{6} + n(1) = \frac{5}{6} + n, (\text{n is any integers})$$

$$n = 0 \rightarrow x = \frac{5}{6}, n = 1 \rightarrow x = \frac{11}{6}, n = 2 \rightarrow x = \frac{17}{6}$$

$$n = -1 \rightarrow x = \frac{-1}{6}, n = -2 \rightarrow x = \frac{-7}{6}$$

Q3. If $f(x) = a\cos(bx + c)$, period of f is π, phase shift of g is $\frac{\pi}{3}$, and $f\left(\frac{3\pi}{2}\right) = -2$, then $f\left(\frac{\pi}{6}\right) =$

Period $\frac{2\pi}{b} = \pi \rightarrow b = 2$, phase shift $-\frac{c}{b} = \frac{\pi}{3} \rightarrow -\frac{c}{2} = \frac{\pi}{3} \rightarrow c = -\frac{2\pi}{3}$

then $f(x) = a\cos\left(2x - \frac{2\pi}{3}\right)$, $-2 = f\left(\frac{3\pi}{2}\right) \rightarrow -2 = a\cos\left(2\frac{3\pi}{2} - \frac{2\pi}{3}\right) = \cos\left(3\pi - \frac{2\pi}{3}\right)$

$-2 = a\cos\left(\pi - \frac{2\pi}{3}\right) = a\cos\frac{\pi}{3} = a\frac{1}{2} \rightarrow -2 = a\frac{1}{2} \rightarrow a = -4$, then $f(x) = -4\cos\left(2x - \frac{2\pi}{3}\right)$

$$f\left(\frac{\pi}{6}\right) = -4s\cos\left(2\frac{\pi}{6} - \frac{2\pi}{3}\right) = -4\cos\left(-\frac{\pi}{3}\right) = -4\left(\frac{1}{2}\right) = -2$$

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Term 051 **Quiz# 3** **Sec.05**

Solutions

Q1. Given $f(x) = -3\cos\left(\frac{2x}{3} - \frac{\pi}{3}\right) - 1$, then

a) The **range** of the function f is

$$[-3-1, 3-1] = [-4, 2]$$

b) The **phase shift** of f is

$$-\frac{c}{b} = -\frac{-\pi/3}{2/3} = \frac{\pi}{2}$$

c) The **period** of f is

$$\frac{2\pi}{2/3} = 3\pi$$

Q2. Given $g(x) = 3\cot\left(\pi x - \frac{\pi}{3}\right) - 5$, find the equation **of vertical asymptotes** of g on the interval $[-2, 3]$

$$g(x) = \frac{3\cos\left(\pi x - \frac{\pi}{3}\right)}{\sin\left(\pi x - \frac{\pi}{3}\right)} - 5$$

$$\sin\left(\pi x - \frac{\pi}{3}\right) = 0 \rightarrow \left(\pi x - \frac{\pi}{3}\right) = 0 \rightarrow \pi x = \frac{\pi}{3} \rightarrow x = \frac{1}{3}$$

$$V.A. \rightarrow x = \frac{1}{3} + n(period) = \frac{1}{3} + n(1) = \frac{1}{3} + n, (\text{n is any integers})$$

$$n = 0 \rightarrow x = \frac{1}{3}, n = 1 \rightarrow x = \frac{4}{3}, n = 2 \rightarrow x = \frac{7}{3}$$

$$n = -1 \rightarrow x = \frac{-2}{3}, n = -2 \rightarrow x = \frac{-5}{3}$$

Q3. If $f(x) = a\cos(bx + c)$, period of f is π, phase shift of g is $\frac{\pi}{3}$, and $f\left(\frac{3\pi}{2}\right) = -2$, then $f\left(\frac{\pi}{6}\right) =$

Period $\frac{2\pi}{b} = \pi \rightarrow b = 2$, phase shift $-\frac{c}{b} = \frac{\pi}{3} \rightarrow -\frac{c}{2} = \frac{\pi}{3} \rightarrow c = -\frac{2\pi}{3}$

then $f(x) = a\cos\left(2x - \frac{2\pi}{3}\right)$, $-2 = f\left(\frac{3\pi}{2}\right) \rightarrow -2 = a\cos\left(2\frac{3\pi}{2} - \frac{2\pi}{3}\right) = \cos\left(3\pi - \frac{2\pi}{3}\right)$

$-2 = a\cos\left(\pi - \frac{2\pi}{3}\right) = a\cos\frac{\pi}{3} = a\frac{1}{2} \rightarrow -2 = a\frac{1}{2} \rightarrow a = -4$, then $f(x) = -4\cos\left(2x - \frac{2\pi}{3}\right)$

$$f\left(\frac{\pi}{6}\right) = -4s\cos\left(2\frac{\pi}{6} - \frac{2\pi}{3}\right) = -4\cos\left(-\frac{\pi}{3}\right) = -4\left(\frac{1}{2}\right) = -2$$

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Term 051 **Quiz# 3** **Sec.09**

Solutions

Q1. Given $f(x) = -3\csc\left(\frac{4x}{3} - \frac{\pi}{3}\right) - 2$, then

a) The **range** of the function f is

$$(-\infty, -3-2] \cup [3-2, \infty) = (-\infty, -5] \cup [1, \infty)$$

b) The **phase shift** of f is

$$-\frac{c}{b} = -\frac{-\pi/3}{4/3} = \frac{\pi}{4}$$

c) The **period** of f is

$$\frac{2\pi}{4/3} = \frac{3\pi}{2}$$

Q2. Given $g(x) = 3\cot\left(\pi x - \frac{\pi}{3}\right) - 5$, find the **x-intercepts** of g on the interval $[-2, 3]$

$$g(x) = \frac{3\cos\left(\pi x - \frac{\pi}{3}\right)}{\sin\left(\pi x - \frac{\pi}{3}\right)} - 5$$

$$\cos\left(\pi x - \frac{\pi}{3}\right) = 0 \rightarrow \left(\pi x - \frac{\pi}{3}\right) = \frac{\pi}{2} \rightarrow \pi x = \frac{5\pi}{6} \rightarrow x = \frac{5}{6}$$

$$x - \text{intercepts} \rightarrow x = \frac{5}{6} + n(period) = \frac{5}{6} + n(1) = \frac{5}{6} + n, (\text{n is any integers})$$

$$n = 0 \rightarrow x = \frac{5}{6}, n = 1 \rightarrow x = \frac{11}{6}, n = 2 \rightarrow x = \frac{17}{6}$$

$$n = -1 \rightarrow x = \frac{-1}{6}, n = -2 \rightarrow x = \frac{-7}{6}$$

Q3. If $f(x) = a\sin(bx + c)$, period of f is π, phase shift of g is $\frac{\pi}{3}$, and $f\left(\frac{3\pi}{2}\right) = -2$, then $f\left(\frac{\pi}{6}\right) =$

Period $\frac{2\pi}{b} = \pi \rightarrow b = 2$, phase shift $-\frac{c}{b} = \frac{\pi}{3} \rightarrow -\frac{c}{2} = \frac{\pi}{3} \rightarrow c = -\frac{2\pi}{3}$

then $f(x) = a\sin\left(2x - \frac{2\pi}{3}\right)$, $-2 = f\left(\frac{3\pi}{2}\right) \rightarrow -2 = a\sin\left(2\frac{3\pi}{2} - \frac{2\pi}{3}\right) = \sin\left(3\pi - \frac{2\pi}{3}\right)$

$-2 = a\sin\left(\pi - \frac{2\pi}{3}\right) = a\sin\frac{\pi}{3} = a\frac{\sqrt{3}}{2} \rightarrow -2 = a\frac{\sqrt{3}}{2} \rightarrow a = -\frac{4}{\sqrt{3}}$, then $f(x) = -\frac{4}{\sqrt{3}}\sin\left(2x - \frac{2\pi}{3}\right)$

$$f\left(\frac{\pi}{6}\right) = -\frac{4}{\sqrt{3}}\sin\left(2\frac{\pi}{6} - \frac{2\pi}{3}\right) = -\frac{4}{\sqrt{3}}\sin\left(-\frac{\pi}{3}\right) = -\frac{4}{\sqrt{3}}\left(\frac{-\sqrt{3}}{2}\right) = 2$$