KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS
MATHEMATICAL DEPARTMENT

Math. 002 Term(042) Quizz7&8

Name: I.D.# Sec.#

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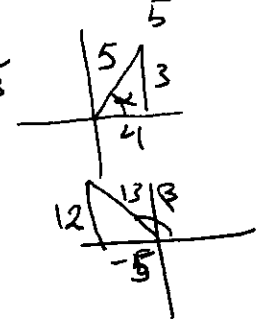
$Q1.\ \sin\left(\sin^{-1}\frac{3}{5}+\cos^{-1}\frac{-5}{13}\right)=$, Let $\alpha=\sin^{-1}\frac{3}{5}\rightarrow \sin\alpha=\frac{3}{5}$

$\beta=\cos^{-1}\frac{-5}{13}\rightarrow\cos\beta=\frac{-5}{13}$



a) $\frac{33}{65}$

b) $\frac{30}{65}$

c) $\frac{32}{65}$

d) $\frac{23}{65}$

e) $\frac{31}{65}$

$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

$=\frac{3}{5}\cdot\frac{-5}{13}+\frac{4}{5}\cdot\frac{12}{13}$

$=\frac{-15+48}{65}=\frac{33}{65}$

$Q2.$ The exact value of $\sin 20^\circ\cos 80^\circ-\sin 70^\circ\cos 10^\circ$ is

a) $\frac{1}{2}$

b) $-\frac{\sqrt{3}}{2}$

c) $-\frac{1}{2}$

d) $\frac{1}{\sqrt{2}}$

e) $-\frac{1}{\sqrt{2}}$

$\cos(90^\circ-20^\circ)\cos 80^\circ-\sin 70^\circ\sin(90^\circ-10^\circ)$

$=\cos 70^\circ\cos 80^\circ-\sin 70^\circ\sin 80^\circ$

$=\cos(70^\circ+80^\circ)=\cos(150^\circ)=-\cos 30^\circ$

$=-\frac{\sqrt{3}}{2}$

$Q3$. The sum of all solutions of the equation

$2\sin x\cos x - 2\sqrt{2}\sin x - \sqrt{3}\cos x + \sqrt{6} = 0$, where $0 \le x < 2\pi$, *is*

a) 2π

b) π

c) -2π

d) $2\pi/3$

e) 3π

$(2\sin x)(\cos x - \sqrt{2}) - \sqrt{3}\cos x + \sqrt{2}\cdot\sqrt{3} = 0$

$(2\sin x)(\cos x - \sqrt{2}) - \sqrt{3}(\cos x - \sqrt{2}) = 0$

$(\cos x - \sqrt{2})(2\sin x - \sqrt{3}) = 0$

$\cos x - \sqrt{2} = 0$ or $2\sin x - \sqrt{3} = 0$

$\cos x = \sqrt{2}$ reject

$\sin x = \frac{\sqrt{3}}{2}$

$\theta' = \frac{\pi}{3}$ in Q I or II

QI $\theta' = \theta \rightarrow x = \frac{\pi}{3}$

QII $\theta' = \pi - \theta \rightarrow \frac{\pi}{3} = \pi - \theta \rightarrow \theta = \frac{2\pi}{3}$

$x = \frac{2\pi}{3}$

Sum $\frac{\pi}{3} + \frac{2\pi}{3} = \pi$

Q4. The exact value of $\cos(112.5^\circ)$ is

a) $\frac{1}{2}$

b) $\frac{\sqrt{2}}{2}$

c) $\frac{\sqrt{2+\sqrt{2}}}{2}$

d) $-\frac{\sqrt{2-\sqrt{2}}}{2}$

e) $-\frac{\sqrt{1-\sqrt{2}}}{2}$

$\cos 112.5^\circ = -\sqrt{\frac{1+\cos 225^\circ}{2}} = -\sqrt{\frac{1-\cos 45^\circ}{2}}$

$= \cos\frac{225}{2}$

$= -\sqrt{\frac{1-\frac{1}{\sqrt{2}}}{2}} = -\sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}}$

$= -\sqrt{\frac{2-\sqrt{2}}{4}} = -\frac{\sqrt{2-\sqrt{2}}}{2}$

$Q5.\ \dfrac{\sin x-\sin 2x}{\cos x+\cos 2x}=$

$a)\cos x$

$b)-\cos x$

$c)-\tan\dfrac{x}{2}$

$d)-\tan x$

$e)-\sin\dfrac{x}{2}$

$$= \frac{\sin x - 2\sin x\cos x}{\cos x + 2\cos^2 x - 1} = \frac{(\sin x)(1-2\cos x)}{2\cos^2 x + \cos x - 1}$$

$$= \frac{(\sin x)(1-2\cos x)}{(2\cos x - 1)(\cos x + 1)} = \frac{-\sin x}{\cos x + 1}$$

$$= -\frac{\sin x}{\cos x + 1} = -\tan\frac{x}{2}$$

$Q6.$ The Domain D and the Range R of the function

$f(x)=2\cos^{-1}(2-3x)+\dfrac{\pi}{2}$ *are*

$a)D=[1/3,1]$ *and* $R=[\pi/2,\ 5\pi/2]$

$b)D=[1/3,1]$ *and* $R=(-\infty,\infty)$

$c)D=[-1,1]$ *and* $R=[\pi/2,\ 5\pi/2]$

$d)D=(-\infty,\infty)$ *and* $R=[\pi/2,\ 5\pi/2]$

$e)D=R=(-\infty,\infty)$

D_f: $-1\le 2-3x\le 1$

$-3\le -3x\le -1$

$1\ge x\ge \frac{1}{3}$

$D_f=[\frac{1}{3},\ 1]$

R_f

$y_1=\cos^{-1}(2-3x)$, $R_{y_1}=[0,\pi]$

$y_2=2\cos^{-1}(2-3x)$, $R_{y_2}=[0,2\pi]$

$\therefore R_f=\left[0+\frac{\pi}{2},\ 2\pi+\frac{\pi}{2}\right]=\left[\frac{\pi}{2},\ \frac{5\pi}{2}\right]$