KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS
MATHEMATICAL DEPARTMENT

Math. 002 Second Test Term(042)

I.D.#

Name:

Sec.#:

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Q1. Find the domain and the range of the function $f(x) = 2\sin^{-1}(2x+1) - 2$

Domain: $-1 \leq 2x+1 \leq 1$

$-2 \leq 2x \leq 0$

$-1 \leq x \leq 0$

$D_f = [-1, 0]$

$y = \sin^{-1} u \rightarrow y \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

$y_1 = 2\sin^{-1} u \rightarrow y_1 \in [-\pi, \pi]$

$f(x) = 2\sin^{-1}(2x+1) - 2$

$f(x) \in [-\pi-2, \pi-2]$

$R_f = [-\pi-2, \pi-2]$

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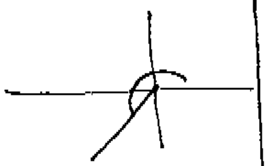
Q2. Find the exact value of $\cos^{-1}\left(\cos\frac{8\pi}{7}\right) = \theta$

$\frac{8\pi}{7} \notin [0, \pi]$



$\theta' = \frac{8\pi}{7} - \pi = \frac{\pi}{7}$

$\cos\frac{8\pi}{7}$ is negative

$\therefore \theta$ in quadrant II

$\therefore \theta' = \pi - \theta$

$\frac{\pi}{7} = \pi - \theta$

$\theta = \pi - \frac{\pi}{7} = \frac{6\pi}{7}$

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Q3. Find the equation of the parabola that has vertex $(-2, 3)$, passes through the point $(2, -3)$, and its axis of symmetry is parallel to the $y-axis$.

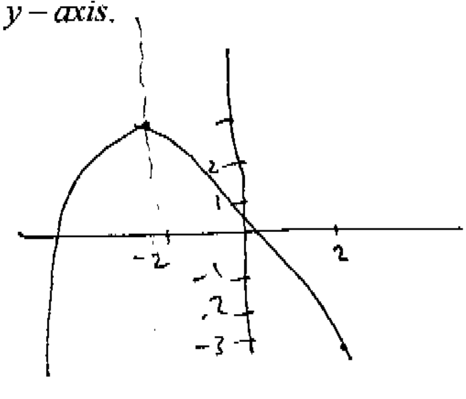
V $(x-h)^2 = 4p(y-k)$

$(x+2)^2 = 4p(y-3)$

point $(2,-3) \Rightarrow (4)^2 = 4p(-6)$

$16 = -24p \Rightarrow p = \frac{16}{-24} = \frac{-2}{3}$

$\therefore$ ~~Eq~~ $(x+2)^2 = 4(\frac{-2}{3})(y-3)$



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Q4. Solve the equation $4\sin^2 x + 2\sqrt{3}\sin x - 2\sin x - \sqrt{3} = 0$, $0 \le x < 2\pi$.

$\Rightarrow 2\sin x\left[2\sin x + \sqrt{3}\right] - \left[2\sin x + \sqrt{3}\right] = 0$

$\left[2\sin x + \sqrt{3}\right]\left[2\sin x - 1\right] = 0$

$\sin x = -\dfrac{\sqrt{3}}{2}$ $\quad$ $\sin x = \dfrac{1}{2}$

$\theta' = \dfrac{\pi}{3}$

Quadrant III $\quad$ Quadrant IV

$\theta = \dfrac{4\pi}{3}$ $\quad$ $\theta = \dfrac{5\pi}{3}$

$\sin x = \frac{1}{2} \rightarrow \theta' = \pi/6$

Quadrant I $\rightarrow \theta = \pi/6$

Quadrant II $\rightarrow \theta = \dfrac{5\pi}{6}$

Solution

$\dfrac{4\pi}{3}, \dfrac{5\pi}{3}, \dfrac{\pi}{6}, \dfrac{5\pi}{6}$

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Q5. Find the equation of the ellipse with foci at $(-2,4)$ *and* $(-2, 2)$ and eccentricity $\dfrac{1}{4}$

V Center $\left(\dfrac{-2+-2}{2}, \dfrac{4+2}{2}\right) = (-2, 3)$

$\dfrac{(x-h)^2}{b^2} + \dfrac{(y-k)^2}{a^2} = 1$

$\boxed{c = 1}$

$e = \dfrac{1}{4} = \dfrac{c}{a} \Rightarrow \dfrac{1}{4} = \dfrac{1}{a} \Rightarrow \boxed{a = 4}$

$c^2 = a^2 - b^2 \Rightarrow 1 = 16 - b^2$

$b^2 = 15$

$\therefore \dfrac{(x+2)^2}{15} + \dfrac{(y-3)^2}{16} = 1$

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Q6. Find the values of k if the following system is inconsistent

$$\begin{array}{r|l} 3 & 2x + 2ky = 3 \\ -2 & 3x - 2y = 0 \end{array}$$

$0 + 6ky + 4y = 6$

$(6k+4)y = 6$

inconsistant $\Rightarrow 6k + 4 = 0$ $\boxed{k = -\dfrac{2}{3}}$

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Q7. Given the vector $v = \langle -2, 3 \rangle$, find the vector u that has magnitude 8 and opposite direction of the vector v.

unit ~~vect~~ vector opposit direction of v is $-\left\langle \frac{-2}{\sqrt{4+9}}, \frac{3}{\sqrt{4+9}} \right\rangle$

$$= \left\langle \frac{2}{\sqrt{13}}, \frac{-3}{\sqrt{13}} \right\rangle$$

$$u = 8 \cdot \left\langle \frac{2}{\sqrt{13}}, \frac{-3}{\sqrt{13}} \right\rangle = \left\langle \frac{16}{\sqrt{13}}, \frac{-24}{\sqrt{13}} \right\rangle$$

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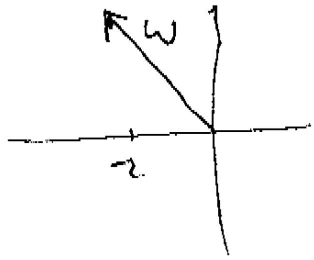
Q8. Find the direction angle of the vector
$w = \langle -2, 2\sqrt{3} \rangle$



$$\tan\theta = \frac{2\sqrt{3}}{-2} = -\sqrt{3}$$

$$\theta' = \frac{\pi}{3} \rightarrow \theta = \frac{2\pi}{3}$$

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Q9. Find the vertices, foci, and asymptotes of the hyperbola
$4x^2 - 9y^2 + 16x + 54y - 29 = 0$

$$4x^2 + 16x - 9y^2 + 54y = 29$$

$$4[x^2 + 4x] - 9[y^2 - 6y] = 29$$

$$4[x^2 + 4x + 2^2 - 2^2] - 9[y^2 - 6y + 3^2 - 3^2] = 29$$

$$4[(x+2)^2 - 4] - 9[(y-3)^2 - 9] = 29$$

$$4(x+2)^2 - 16 - 9(y-3)^2 + 81 = 29$$

$$4(x+2)^2 - 9(y-3)^2 = -36$$

$$-\frac{(x+2)^2}{9} + \frac{(y-3)^2}{4} = 1$$

$$\frac{(y-3)^2}{4} - \frac{(x+2)^2}{9} = 1$$

V. center $(-2, 3)$

$b^2 = 9 \rightarrow \boxed{b = 3}$, $a^2 = 4 \rightarrow \boxed{a = 2}$

v

vertices $(-2, 3 \pm 2) \Rightarrow (-2, 5), (-2, 1)$

$c^2 = a^2 + b^2 = 13$ foci $(-2, 3 \pm \sqrt{13})$

Asymptotes:

$$\frac{(y-3)^2}{4} - \frac{(x+2)^2}{9} = 0$$

$$\frac{(y-3)^2}{4} = \frac{(x+2)^2}{9} \Rightarrow (y-3)^2 = \frac{4}{9}(x+2)^2$$

$$y - 3 = \pm \frac{2}{3}(x+2)$$