King Fahd University of Petroleum &Minerals
Department of Mathematical Sciences
Math 101 5, 9 & 13

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Second Major Exam Semester 032
Time: 6:30-7:30 pm, Saturday 10.4.2004

Name: ID : Ser.#: Section :

Show all your work. No credits for answers not supported by work

Q1.(6 points) Use the table to find

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	2	−2	3	1
1	0	4	1	0
2	5	−1	1	3

a) $h'(1)$ if $h(x) = f(g(2x))$

$h'(x) = f'(g(2x)) \cdot g'(2x) \cdot 2$

$h'(1) = f'(g(2)) \cdot g'(2) \cdot 2 = f'(1) \cdot 3 \cdot 2 = 6(4) = 24$

b) $F'(0)$ if $F(x) = \dfrac{f(x)}{4+g(x)}$

$$F'(x) = \frac{[4+g(x)] \cdot f'(x) - f(x) \cdot g'(x)}{[4+g(x)]^2}$$

$$F'(0) = \frac{[4+g(0)] \cdot f'(0) - f(0)\, g'(0)}{[4+g(0)]^2} = \frac{[4+3] \cdot (-2) - (2)(1)}{[4+3]^2} = \frac{-14-2}{49}$$

$$= \frac{-16}{49}$$

Q2.(7 points) Find $\dfrac{d^2y}{dx^2}$ if $x - xy + y = 1$ (write your answer in simplest form)

$$1 - \left(x\frac{dy}{dx} + (1)y\right) + \frac{dy}{dx} = 0 \Rightarrow (1-x)\frac{dy}{dx} = y-1 \Rightarrow \boxed{\frac{dy}{dx} = \frac{y-1}{1-x}}$$

$$\frac{d^2y}{dx^2} = \frac{(1-x)(dy/dx) - (y-1)(-1)}{(1-x)^2} = \frac{\cancel{(1-x)}\frac{y-1}{\cancel{(1-x)}} + y - 1}{(1-x)^2}$$

$$= \frac{2y-2}{(1-x)^2}$$

Q3. Find the equation of the tangent line to the curve $x + x^2\cos(y+\pi) + \left(y - \frac{\pi}{2}\right)^2 = 1$

at the point $\left(1, \frac{\pi}{2}\right)$

$$1 - \left(2x\cos(y+\pi) + x^2\sin(y+\pi)\frac{dy}{dx}\right) + 2\left(y - \frac{\pi}{2}\right)\cdot\frac{dy}{dx} = 0$$

at $(1, \frac{\pi}{2}) \Rightarrow 1 - \left(2\underbrace{\cos\left(\frac{3\pi}{2}\right)}_{0} - 1\sin\left(\frac{3\pi}{2}\right)\frac{dy}{dx}\right) + 0 = 0$

$$1 + \frac{dy}{dx} = 0 \rightarrow \left.\frac{dy}{dx}\right|_{(1,\frac{\pi}{2})} = 1$$

$$y - \frac{\pi}{2} = 1(x-1)$$

Q4. Use Local Linear Approximation Method to estimate $\cos(62^\circ)$.

$y = \cos x$; $x_0 = 60^\circ = \frac{\pi}{3}$ rad. , $62^\circ = 60^\circ + 2^\circ = \frac{\pi}{3} + \frac{2\pi}{180} = \frac{\pi}{3} + \frac{\pi}{90}$ rad.

$\cos x \approx f(\frac{\pi}{3}) + f'(\frac{\pi}{3})(x - \frac{\pi}{3})$, $\cos\frac{\pi}{3} = \frac{1}{2}$, $y' = -\sin x$

$f'(\frac{\pi}{3}) = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$

$\cos x \approx \frac{1}{2} - \frac{\sqrt{3}}{2}(x - \frac{\pi}{3})$

$\cos 62^\circ \approx \frac{1}{2} - \frac{\sqrt{3}}{2}\left(\frac{\pi}{3} + \frac{\pi}{90} - \frac{\pi}{3}\right) = \frac{1}{2} - \frac{\sqrt{3}}{2}\left(\frac{\pi}{90}\right) = \frac{1}{2} - \frac{\sqrt{3}\pi}{180}$

Q5. At what points is the tangent line to the curve $y = 2x^3 - 8x + 1$ perpendicular to the line $2y - x + 2 = 0$?

$\frac{dy}{dx} = 6x^2 - 8$, slope of the line $2y - x + 2 = 0$ is $\frac{1}{2}$

$\therefore \perp \Rightarrow (6x^2 - 8)(\frac{1}{2}) = -1 \Rightarrow 6x^2 - 8 = -2 \Rightarrow x = \pm 1$

$\Rightarrow x = 1 \Rightarrow y = 1(1)^3 - 8(1) + 1 = -6 \Rightarrow (1, -6)$

$x = -1 \Rightarrow y = -2 + 8 + 1 = 7 \Rightarrow (-1, 7)$

Q6. Given $f(x)=\frac{x}{x+1}$. Use the **definition** of the derivative to find $f'(x)$

$$f'(x)=\lim_{w\to x}\frac{\frac{w}{w+1}-\frac{x}{x+1}}{w-x}=\lim_{w\to x}\frac{\frac{w(x+1)-x(w+1)}{(w+1)(x+1)}}{w-x}$$

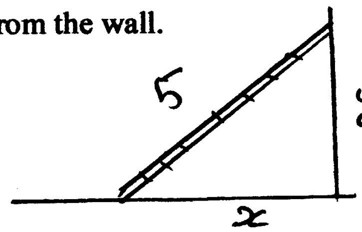
$$=\lim_{w\to x}\frac{wx+w-xw-x}{(w+1)(x+1)(w-x)}=\lim_{w\to x}\frac{w-x}{(w+1)(x+1)(w-x)}$$

$$=\lim_{w\to x}\frac{1}{(w+1)(x+1)}=\frac{1}{(x+1)(x+1)}=\frac{1}{(x+1)^2}$$

Q7. A 5-m ladder is leaning against a wall. If the foot of the ladder pulled away from the wall at the rate of $2\,m/\min$.

(a) How fast will the top sliding down the wall when the foot is 3-m from the wall.

(b) At what rate is the area of the triangle formed when the foot is 3-m from the wall.



(a)

$$\frac{dx}{dt}=2\text{ m/min}. \qquad \left.\frac{dy}{dt}\right|_{x=3} \;??$$

$$25=y^2+x^2 \Rightarrow 0=2y\frac{dy}{dt}+2x\frac{dx}{dt}$$

$$25=y^2+3^2 \Rightarrow \underline{y=4}$$

$$\therefore\; 0=2\cdot4\cdot\frac{dy}{dt}+2\cdot3(2) \Rightarrow \frac{dy}{dt}=\frac{-12}{8}=\frac{-3}{2}\text{ m/min}$$

(b)

$$A=\frac{1}{2}xy \Rightarrow \frac{dA}{dt}=\frac{1}{2}x\frac{dy}{dt}+\frac{1}{2}\frac{dx}{dt}y$$

$$=\frac{1}{2}\cdot3\cdot\frac{-3}{2}+\frac{1}{2}\cdot2\cdot4$$

$$=\frac{-9}{2}+4=-\frac{1}{2}\text{ m}^2/\text{min}$$