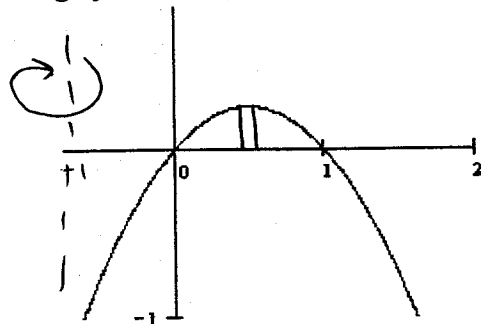


King Fahd University of Petroleum & Minerals
Department of Mathematical Sciences
Math 102-01 Quiz # 2

Name: Sec. # 1 I.D. # _____ Sr.# _____

1. Set up (do not evaluate) the integral for the volume of the solid obtained by rotating the region bounded by $y = x - x^2$ and $y = 0$ about the line $x = -1$ (by using cylindrical shell method).

$$V = \int_0^1 2\pi(x+1)(x-x^2) dx$$



2. Find the arc length of the curve $y = x^{2/3}$ from (1,1) to (8,4).

$$\frac{dy}{dx} = \frac{2}{3} x^{-1/3} \quad 1 + \left[\frac{dy}{dx} \right]^2 = 1 + \frac{4x^{-2/3}}{9} = 1 + \frac{4}{9x^{2/3}}$$

$$= \frac{9x^{2/3} + 4}{9x^{2/3}}$$

$$L = \int_1^8 \frac{\sqrt{9x^{2/3} + 4}}{3x^{1/3}} dx, \quad u = 9x^{2/3} + 4 \rightarrow du = 6x^{-1/3} dx$$

$$L = \int_{13}^{40} \frac{\sqrt{u}}{18} du = \frac{1}{18} \left[\frac{u^{3/2}}{3/2} \right]_{13}^{40} = \frac{1}{27} \left[40^{3/2} - 13^{3/2} \right]$$

3. Evaluate $\int \sinh^4 x \cosh^3 x dx = I$

$$\int \sinh^4 x \cosh^2 x \cosh x dx = \int \sinh^4 x (1 + \sinh^2 x) \cosh x dx$$

$$u = \sinh x \rightarrow du = \cosh x dx$$

$$I = \int u^4 (1 + u^2) du = \int (u^4 + u^6) du = \frac{u^5}{5} + \frac{u^7}{7} + C$$

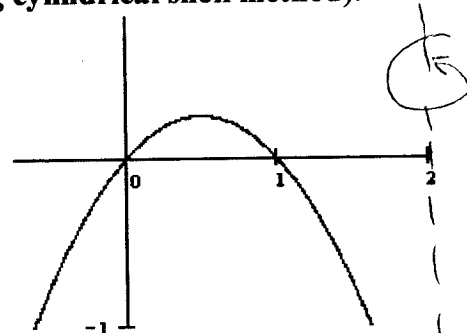
$$= \frac{\sinh^5 x}{5} + \frac{\sinh^7 x}{7} + C$$

King Fahd University of Petroleum & Minerals
Department of Mathematical Sciences
Math 102-02 Quiz # 2

Name: Sec. #2 I.D. # _____ Sr.# _____

1. Set up (do not evaluate) the integral for the volume of the solid obtained by rotating the region bounded by $y = x - x^2$ and $y = 0$ about the line $x = 2$ (by using cylindrical shell method).

$$V = \int_0^1 2\pi(2-x)(x-x^2) dx$$



2. Find the arc length of the curve $y = x^{2/3}$ from (1,1) to (8,4).

$$\frac{dy}{dx} = \frac{2}{3} x^{-1/3} \quad 1 + \left(\frac{dy}{dx}\right)^2 = 1 + \frac{4}{9} x^{-2/3} = 1 + \frac{4}{9x^{2/3}}$$

$$L = \int_1^8 \sqrt{\frac{9x^{2/3} + 4}{9x^{2/3}}} dx = \int_1^8 \frac{\sqrt{9x^{2/3} + 4}}{3x^{1/3}} dx$$

$$u = 9x^{2/3} + 4 \rightarrow du = 6x^{-1/3} dx = \frac{1}{18} \int_{13}^{40} \sqrt{u} du$$

$$= \frac{1}{18} \left[\frac{u^{3/2}}{3/2} \right]_{13}^{40} = \frac{1}{18} \cdot \frac{2}{3} \left[40^{3/2} - (13)^{3/2} \right]$$

$$= \frac{1}{27} \left[40^{3/2} - 13^{3/2} \right]$$

3. Evaluate $\int \frac{\coth^3 x}{\sinh^3 x} dx = \int \frac{\cosh^3 x}{\sinh^3 x \sinh^3 x} dx = \int \frac{\cosh^3 x}{\sinh^6 x} dx$

$$= \int \frac{\cosh x (1 + \sinh^2 x)}{\sinh^6 x} dx, \quad u = \sinh x \rightarrow du = \cosh x dx$$

$$\int \frac{1+u^2}{u^6} du = \int \frac{1}{u^6} + \frac{1}{u^4} du = \frac{u^{-5}}{-5} + \frac{u^{-3}}{-3} + C$$

$$\frac{1}{\sinh^5 x} - \frac{1}{3\sinh^3 x} + C$$