

Q1. Find x-coordinate of the points so that the graph of $f(x) = \frac{x^2+1}{x+1}$ has **horizontal** tangent line.

$$0 = f'(x) = \frac{(x+1)(2x) - (x^2+1)(1)}{(x+1)^2} = \frac{2x^2+2x-x^2-1}{(x+1)^2}$$

$$= \frac{x^2+2x-1}{(x+1)^2} = 0 \Rightarrow x^2+2x-1=0$$

$$x = \frac{-2 \pm \sqrt{4-4(1)(-1)}}{2} = \frac{-2 \pm \sqrt{8}}{2} = \frac{-2 \pm 2\sqrt{2}}{2}$$

$$x = -1 \pm \sqrt{2}$$

Q2. Let $f(x) = \cos x$. Use the **definition** of the derivative to find $f'(x)$.

$$f'(x) = \lim_{h \rightarrow 0} \frac{\cos(x+h) - \cos x}{h} = \lim_{h \rightarrow 0} \frac{\cos h \cos x - \sin h \sin x - \cos x}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\cos x (\cos h - 1) - \sin h \sin x}{h} = \lim_{h \rightarrow 0} \frac{\cos x (\cos h - 1)}{h} - \lim_{h \rightarrow 0} \frac{\sin h \sin x}{h}$$

$$= \cos x \lim_{h \rightarrow 0} \frac{\cos h - 1}{h} - \sin x \lim_{h \rightarrow 0} \frac{\sin h}{h}$$

$$= (\cos x)(0) - (\sin x)(1) = -\sin x$$

Q3. Find k if the curve $y = 2x^2 - 3x - 2$ is tangent to the line $y = 5x - 2k$.

$$\text{slope} = \frac{dy}{dx} = 4x - 3 \quad \left. \begin{array}{l} \Rightarrow 5 = 4x - 3 \\ \Rightarrow x = 2 \end{array} \right\}$$

$$\text{slope of } y = 5x - 2k \text{ is } 5$$

$$\therefore y = 2(2)^2 - 3(2) - 2 = 8 - 6 - 2 = 0$$

$$\text{The tangent point is } (2, 0) \Rightarrow 0 = 5(2) - 2k$$

$$\Rightarrow 2k = 10 \Rightarrow k = 5$$