

Quiz 2

I.D.#:

Ser.#:

Name:

Q1. Find x-coordinate of the points so that the graph of $f(x) = \frac{x^2+1}{x-1}$ has **horizontal** tangent line.

$$f'(x) = \frac{(x-1)(2x) - (x^2+1)(1)}{(x-1)^2} = \frac{2x^2 - 2x - x^2 - 1}{(x-1)^2} = \frac{x^2 - 2x - 1}{(x-1)^2}$$

$$f'(x) = 0 \Rightarrow x^2 - 2x - 1 = 0 \Rightarrow x = \frac{2 \pm \sqrt{4 - 4(1)(-1)}}{2} = \frac{2 \pm \sqrt{8}}{2}$$

$$\therefore x = \frac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2}$$

Q2. Let $f(x) = \sqrt{1-3x}$. Use the **definition** of the derivative to find $f'(x)$.

$$f'(x) = \lim_{h \rightarrow 0} \frac{\sqrt{1-3(x+h)} - \sqrt{1-3x}}{h} = \lim_{h \rightarrow 0} \frac{\sqrt{1-3(x+h)} - \sqrt{1-3x}}{h} \cdot \frac{\sqrt{1-3(x+h)} + \sqrt{1-3x}}{\sqrt{1-3(x+h)} + \sqrt{1-3x}}$$

$$= \lim_{h \rightarrow 0} \frac{1-3(x+h) - (1-3x)}{h(\sqrt{1-3(x+h)} + \sqrt{1-3x})} = \lim_{h \rightarrow 0} \frac{-3h}{h(\sqrt{1-3(x+h)} + \sqrt{1-3x})}$$

$$= \lim_{h \rightarrow 0} \frac{-3}{\sqrt{1-3(x+h)} + \sqrt{1-3x}} = \frac{-3}{\sqrt{1-3x} + \sqrt{1-3x}} = \frac{-3}{2\sqrt{1-3x}}$$

Q3. Find k if the curve $y = 2x^2 + 3x - 2$ is tangent to the line $y = 11x - 4k$.

$$\text{slope} = \frac{dy}{dx} = 4x + 3$$

$$\text{slope of } y = 11x - 4k \text{ is } 11 \quad \Rightarrow \quad 4x + 3 = 11 \Rightarrow x = 2$$

$$y = 2(2)^2 + 3(2) - 2 = 8 + 6 - 2 = 12$$

$$\text{The tangent point is } (2, 12) \Rightarrow 12 = 11(2) - 4k$$

$$\Rightarrow 4k = 22 - 12 = 10$$

$$k = \frac{10}{4} = \frac{5}{2}$$