King Fahd University of Petroleum &Minerals
Department of Mathematical Sciences
Math 102 1 & 2

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Second Major Exam Semester 033
Time: 4:30-5:45 pm, Sat. 31.7.2004

Name: ID : Ser.#: Section :

Show all your work. No credits for answers not supported by work

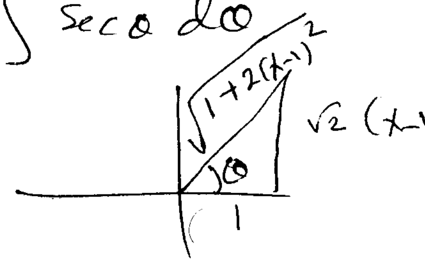
Q1. Evaluate $\int \frac{dx}{\sqrt{2x^2-4x+3}}$

$$= \int \frac{dx}{\sqrt{2[x^2-2x]+3}} = \int \frac{dx}{\sqrt{2[x^2-2x+1-1]+3}} = \int \frac{dx}{\sqrt{2[(x-1)^2-1]+3}}$$

$$= \int \frac{dx}{\sqrt{2(x-1)^2-2+3}} = \int \frac{dx}{\sqrt{2(x-1)^2+1}} = \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{(x-1)^2+\frac{1}{2}}}$$

Let $x-1 = \frac{1}{\sqrt{2}} \tan\theta \rightarrow dx = \frac{1}{\sqrt{2}} \sec^2\theta \, d\theta$

$$= \frac{1}{2} \int \frac{\sec^2\theta \, d\theta}{\sqrt{\frac{1}{2}\tan^2\theta + \frac{1}{2}}} = \frac{\sqrt{2}}{2} \int \frac{\sec^2\theta \, d\theta}{\sqrt{\sec^2\theta}} = \frac{\sqrt{2}}{2} \int \sec\theta \, d\theta$$

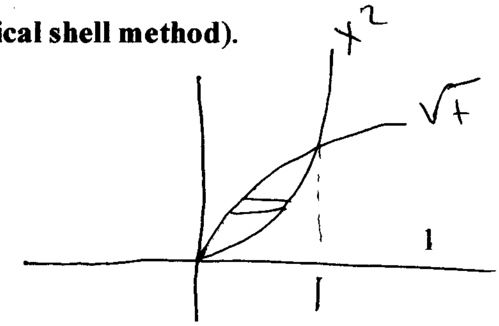


$$= \frac{\sqrt{2}}{2} \ln\left|\sec\theta + \tan\theta\right| + C$$

$$= \frac{\sqrt{2}}{2} \ln\left|\sqrt{1+2(x-1)^2} + \sqrt{2}(x-1)\right| + C$$

Q2. Set up (**Do Not Evaluate**) the integral for the volume of the solid obtained by rotating the region bounded by $y=\sqrt{x}$ and $y=x^2$ about the line $y=-1$ (**by using cylindrical shell method**).

$$V = \int_0^1 2\pi(y+1)\left(\sqrt{y}-y^2\right) dy$$



Q3. Evaluate $\int_1^4 e^{\sqrt{x}}\,dx$

Let $w = \sqrt{x}$ $\rightarrow dw = \frac{1}{2\sqrt{x}}\,dx \rightarrow dx = 2w\,dw$

$$= \int_1^2 e^w \cdot 2w\,dw = 2\int_1^2 e^w\, w\,dw$$ Now by part.

$u = w \qquad dv = e^w\,dw$

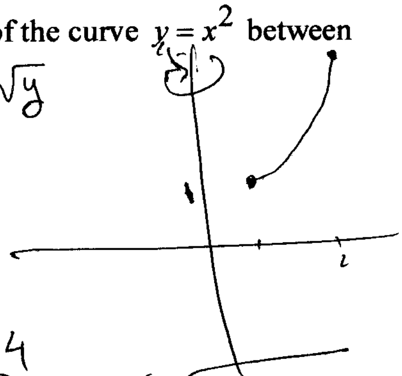
$du = dw \qquad v = e^w$

$$I = 2\left[we^w\Big|_1^2 - \int_1^2 e^w\,dw \right] = 2\left[2e^2 - e - \left[e^w\Big|_1^2 \right] \right]$$

$$= 2\left[2e^2 - e - (e^2 - e) \right] = 2\left[e^2 \right] = 2e^2$$

Q4. Find the area of the surface that is generated by revolving the portion of the curve $y = x^2$ between $x = 1$ and $x = 2$ about the y-axis.

$\cancel{\frac{dx}{dy}} =$ $\quad x = \sqrt{y}$



$$S = \int_1^4 2\pi \sqrt{y}\, \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$= 2\pi \int_1^4 \sqrt{y}\, \sqrt{1 + \left(\frac{1}{2\sqrt{y}}\right)^2}\, dy$$

$$= 2\pi \int_1^4 \sqrt{y}\, \sqrt{1 + \frac{1}{4y}}\, dy = 2\pi \int_1^4 \cancel{\sqrt{y}}\, \frac{\sqrt{4y+1}}{2\cancel{\sqrt{y}}}\, dy$$

$$= \pi \int_1^4 \sqrt{4y+1}\, dy \qquad \boxed{\begin{array}{l} u = 4y+1 \\ du = 4\,dy \end{array}}$$

$$= \frac{\pi}{4} \int_5^{17} u^{\frac{1}{2}}\, du$$

$$= \frac{\pi}{4} \cdot \frac{2}{3} u^{3/2} \Big|_5^{17} = \frac{\pi}{6} \left[17^{3/2} - 5^{3/2} \right]$$

Q5. Evaluate $\int_0^{63} \frac{dx}{\sqrt{x+1}+\sqrt[3]{x+1}}$, $u = (x+1)^{1/6} \to du = \frac{1}{6}(x+1)^{-5/6}\,dx$

$\to dx = 6(x+1)^{5/6}\,du = 6u^5\,du$

$$= \int_1^2 \frac{6u^5\,du}{u^3+u^2} = 6\int_1^2 \frac{u^3}{u+1}\,du$$

$$\begin{array}{r} u^2-u+1 \\ u+1 \overline{\big)\ u^3 \qquad\qquad} \\ -u^3 \mp u^2 \\ \hline -u^2 \\ \mp u^2 \pm u \\ \hline u \\ -u \mp 1 \\ \hline -1 \end{array}$$

$$= 6\int_1^2 \left(u^2-u+1-\frac{1}{u+1}\right)du$$

$$= 6\left[\frac{u^3}{3}-\frac{u^2}{2}+u-\ln|u+1|\right]_1^2$$

$$= 6\left\{\frac{8}{3}-\frac{4}{2}+2-\ln|3|-\left(\frac{1}{3}-\frac{1}{2}+1-\ln 2\right)\right\}$$

$$= 6\left\{\frac{8}{3}-\ln|3|-\left(\frac{5}{6}-\ln 2\right)\right\}$$

$$= 6\left[\frac{8}{3}-\frac{5}{6}-\ln 3+\ln 2\right] = 8-5-6\left(\ln\frac{2}{3}\right) = 3-6\ln\frac{2}{3}$$

Q6. Evaluate $\int_0^1 x\tan^{-1}x\,dx = I$

$u = \tan^{-1}x \qquad dv = x\,dx$

$du = \frac{1}{1+x^2}\,dx \qquad v = x^2$

$$\begin{array}{r} 1 \\ 1+x^2 \overline{\big)\ x^2 \qquad} \\ -1 \mp x^2 \\ \hline -1 \end{array}$$

$$I = x^2\tan^{-1}x\Big]_0^1 - \int_0^1 \frac{x^2}{1+x^2}\,dx = \tan^{-1}1 - \int_0^1\left(1-\frac{1}{1+x^2}\right)dx$$

$$= \frac{\pi}{4} - \left\{x-\tan^{-1}x\right\}_0^1 = \frac{\pi}{4} - \left\{1-\tan^{-1}1-(0)\right\}$$

$$= \frac{\pi}{4}-1+\tan^{-1}1 = \frac{\pi}{4}-1+\frac{\pi}{4} = \frac{\pi}{2}+1$$

Q7. Evaluate $\int \tan^5 x \sec^5 x\, dx$

$$= \int \tan^4 x \sec^4 x \tan x \sec x \, dx = \int (\sec^2 x - 1)^2 \sec^4 x \tan x \sec x \, dx$$

$$= \int (u^2-1)^2 u^4 \, du \Leftarrow \boxed{\begin{array}{l} u = \sec x \\ du = \sec x \tan x \, dx \end{array}}$$

$$= \int (u^8 - 2u^6 - u^4) \, du =$$

$$= \frac{u^9}{9} - \frac{2u^7}{7} - \frac{u^5}{5} + c$$

$$= \frac{\sec^9 x}{9} - \frac{2\sec^7 x}{7} - \frac{\sec^5 x}{5} + c$$

Q8. Evaluate $\int \frac{x^3 - 2x + 1}{x^4 + x^2} dx$. $= I$

$$\frac{x^3 - 2x + 1}{x^4 + x^2} = \frac{x^3 - 2x + 1}{x(x^2+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx + D}{x^2+1}$$

$$= \frac{Ax(x^2+1) + B(x^2+1) + (Cx+D)(x^2+1)}{x^2(x^2+1)}$$

$$\Rightarrow x^3 - 2x + 1 = Ax(x^2+1) + B(x^2+1) + (Cx+D)(x^2+1)$$

$$= (A+C)x^3 + (B+D)x^2 + Ax + B$$

constant $\rightarrow \boxed{1 = B}$, coeff. of $x \rightarrow \boxed{-2 = A}$

coeff $x^3 \rightarrow 1 = A + C \rightarrow \boxed{C = 3}$, coeff. of $x^2 \rightarrow 0 = B + D \rightarrow \boxed{D = -1}$

$$\Rightarrow I = \int \left[\frac{-2}{x} + \frac{1}{x^2} + \frac{3x-1}{x^2+1}\right] dx$$

$$= -2\ln|x| - \frac{1}{x} + \int \frac{3x}{x^2+1} dx + \int \frac{1}{x^2+1} dx$$

$$= 2\ln|x| - \frac{1}{x} + 3\ln(x^2+1) - \tan^{-1} x + c$$