

King Fahd University of Petroleum & Minerals
 Department of Mathematical Sciences
 Math 102 1 & 2

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 Second Major Exam Semester 033
 Time: 4:30-5:45 pm, Sat. 31.7.2004

Name: _____ ID: _____ Ser.#: _____ Section: _____
 Show all your work. No credits for answers not supported by work

Q1. Evaluate $\int \frac{dx}{\sqrt{2x^2 - 4x + 3}}$

$$= \int \frac{dx}{\sqrt{2[x^2 - 2x] + 3}} = \int \frac{dx}{\sqrt{2[x^2 - 2x + 1 - 1] + 3}} = \int \frac{dx}{\sqrt{2[(x-1)^2 - 1] + 3}}$$

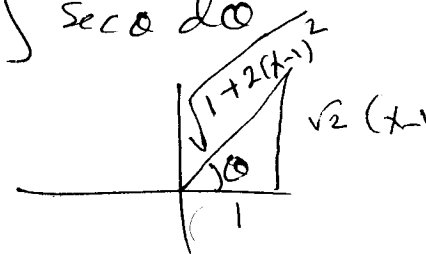
$$= \int \frac{dx}{\sqrt{2(x-1)^2 - 2 + 3}} = \int \frac{dx}{\sqrt{2(x-1)^2 + 1}} = \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{(x-1)^2 + \frac{1}{2}}}$$

Let $x-1 = \frac{1}{\sqrt{2}} \tan \theta \rightarrow dx = \frac{1}{\sqrt{2}} \sec^2 \theta d\theta$

$$= \frac{1}{2} \int \frac{\sec^2 \theta d\theta}{\sqrt{\frac{1}{2} \tan^2 \theta + \frac{1}{2}}} = \frac{\sqrt{2}}{2} \int \frac{\sec^2 \theta d\theta}{\sqrt{\sec^2 \theta}} = \frac{\sqrt{2}}{2} \int \sec \theta d\theta$$

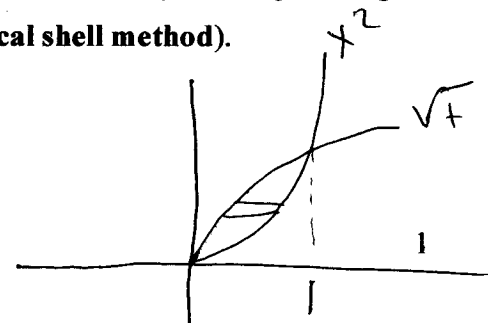
$$= \frac{\sqrt{2}}{2} \ln |\sec \theta + \tan \theta| + C$$

$$= \frac{\sqrt{2}}{2} \ln |\sqrt{1 + 2(x-1)^2} + \sqrt{2}(x-1)| + C$$



Q2. Set up (Do Not Evaluate) the integral for the volume of the solid obtained by rotating the region bounded by $y = \sqrt{x}$ and $y = x^2$ about the line $y = -1$ (by using cylindrical shell method).

$$V = \int_0^1 2\pi(y+1)(\sqrt{y} - y^2) dy$$



Q3. Evaluate $\int_1^4 e^{\sqrt{x}} dx$

Let $u = \sqrt{x} \rightarrow du = \frac{1}{2\sqrt{x}} dx \rightarrow dx = 2u du$

$= \int_1^2 e^u \cdot 2u du$ Now by part.

$u = w \quad dv = e^w dw$

$du = dw \quad v = e^w$

$$I = 2 \left[w e^w \Big|_1^2 - \int_1^2 e^w dw \right] = 2 \left[2e^2 - e - [e^w]_1^2 \right]$$

$$= 2 \left[2e^2 - e - (e^2 - e) \right] = 2 \left[e^2 \right] = 2e^2$$

Q4. Find the area of the surface that is generated by revolving the portion of the curve $y = x^2$ between $x=1$ and $x=2$ about the y -axis.

$\frac{dx}{dy} = x = \sqrt{y}$

$$S = \int_1^4 2\pi \sqrt{y} \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$$

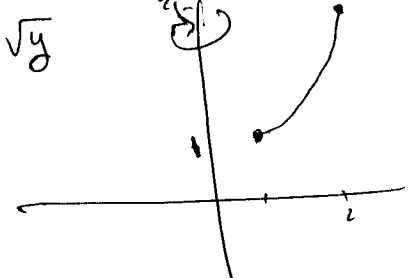
$$= 2\pi \int_1^4 \sqrt{y} \sqrt{1 + \left(\frac{1}{2\sqrt{y}}\right)^2} dy$$

$$= 2\pi \int_1^4 \sqrt{y} \sqrt{1 + \frac{1}{4y}} dy = 2\pi \int_1^4 \frac{\sqrt{4y+1}}{2\sqrt{y}} dy$$

$$= \pi \int_1^4 \sqrt{4y+1} dy$$

$u = 4y + 1$
 $du = 4 dy$

$$= \frac{\pi}{4} \int_5^{17} u^{\frac{1}{2}} du$$

$$= \frac{\pi}{4} \cdot \frac{2}{3} u^{\frac{3}{2}} \Big|_5^{17} = \frac{\pi}{6} \left[17^{\frac{3}{2}} - 5^{\frac{3}{2}} \right]$$


Q5. Evaluate $\int_0^6 \frac{dx}{\sqrt{x+1} + \sqrt[3]{x+1}}$, $u = (x+1)^{\frac{1}{6}} \rightarrow du = \frac{1}{6} (x+1)^{-\frac{5}{6}} dx$

$\rightarrow dx = 6 (x+1)^{\frac{5}{6}} du = 6 u^5 du$

$= \int_1^2 \frac{6 u^5 du}{u^3 + u^2} = 6 \int_1^2 \frac{u}{u+1} du$

$= 6 \int_1^2 \left(u^2 - u + 1 - \frac{1}{u+1} \right) du$

$= 6 \left[\frac{u^3}{3} - \frac{u^2}{2} + u - \ln|u+1| \right]_1^2$

$= 6 \left[\frac{8}{3} - \frac{4}{2} + 2 - \ln|3| - \left(\frac{1}{3} - \frac{1}{2} + 1 - \ln 2 \right) \right]$

$= 6 \left[\frac{8}{3} - \ln|3| - \left(\frac{5}{6} - \ln 2 \right) \right]$

$= 6 \left[\frac{8}{3} - \frac{5}{6} - \ln 3 + \ln 2 \right] = 8 - 5 - 6 \left(\ln \frac{2}{3} \right) = 3 - 6 \ln \frac{2}{3}$

$$\begin{array}{r} u+1 \overline{) \begin{array}{r} u^2 - u + 1 \\ -u^3 + u^2 \\ \hline -u^2 + u \\ -u^2 + u \\ \hline u \\ -u + 1 \\ \hline -1 \end{array}} \end{array}$$

Q6. Evaluate $\int_0^1 x \tan^{-1} x dx = \underline{\underline{\pi}}$

$u = \tan^{-1} x \quad dv = x dx$

$du = \frac{1}{1+x^2} dx \quad v = \frac{x^2}{2}$

$\int = x^2 \tan^{-1} x \Big|_0^1 - \int_0^1 \frac{x^2}{1+x^2} dx = \tan^{-1} 1 - \int_0^1 \left(1 - \frac{1}{1+x^2} \right) dx$

$= \frac{\pi}{4} - \left[x - \tan^{-1} x \right]_0^1 = \frac{\pi}{4} - \left[1 - \tan^{-1} 1 - (0) \right]$

$= \frac{\pi}{4} - 1 + \tan^{-1} 1 = \frac{\pi}{4} - 1 + \frac{\pi}{4} = \frac{\pi}{2} - 1$

$$1+x^2 \overline{) \begin{array}{r} x^2 \\ -1+x^2 \\ \hline -1 \end{array}}$$

Q7. Evaluate $\int \tan^5 x \sec^5 x dx$

$$\begin{aligned}
 &= \int \tan^4 x \sec^4 x \tan x \sec x dx = \int (\sec^2 x - 1)^2 \sec^4 x \tan x \sec x dx \\
 &= \int (u^2 - 1)^2 u^4 du \quad \leftarrow \begin{array}{l} u = \sec x \\ du = \sec x \tan x dx \end{array} \\
 &= \int (u^8 - 2u^6 - u^4) du = \\
 &= \frac{u^9}{9} - \frac{2u^7}{7} - \frac{u^5}{5} + C \\
 &= \frac{\sec^9 x}{9} - \frac{2\sec^7 x}{7} - \frac{\sec^5 x}{5} + C
 \end{aligned}$$

Q8. Evaluate $\int \frac{x^3 - 2x + 1}{x^4 + x^2} dx = I$

$$\begin{aligned}
 \frac{x^3 - 2x + 1}{x^4 + x^2} &= \frac{x^3 - 2x + 1}{x(x^2 + 1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx + D}{x^2 + 1} \\
 &= \frac{Ax(x^2 + 1) + B(x^2 + 1) + (Cx + D)(x^2 + 1)}{x^2(x^2 + 1)}
 \end{aligned}$$

$$\Rightarrow x^3 - 2x + 1 = Ax(x^2 + 1) + B(x^2 + 1) + (Cx + D)(x^2 + 1)$$

$$= (A + C)x^3 + (B + D)x^2 + Ax + B$$

constant $\rightarrow 1 = B$, coeff. of $x \rightarrow -2 = A$

coeff $x^3 \rightarrow 1 = A + C \rightarrow C = 3$, coeff. of $x^2 \rightarrow 0 = B + D \rightarrow D = -1$

$$\begin{aligned}
 \Rightarrow I &= \int \left[\frac{-2}{x} + \frac{1}{x^2} + \frac{3x - 1}{x^2 + 1} \right] dx \\
 &= -2 \ln|x| - \frac{1}{x} + \int \frac{3x}{x^2 + 1} dx + \int \frac{-1}{x^2 + 1} dx \\
 &= 2 \ln|x| - \frac{1}{x} + 3 \ln|x^2 + 1| - \tan^{-1} x + C
 \end{aligned}$$