KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS
MATHEMATICAL DEPARTMENT

Math. 002&004 Test#1 Term(042)

Name: I.D.# Sec.#

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Q1. Find the values of x and y such that $W\left(-\frac{44\pi}{3}\right)=P(x,y)$.

$\frac{44\pi}{3}=14\pi+\frac{2\pi}{3}$ so $\frac{44\pi}{3}$ *and* $\frac{2\pi}{3}$ are corerminal angles

$x=\cos\left(\frac{-44\pi}{3}\right)=\cos\left(\frac{44\pi}{3}\right)$ even function

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$=\cos\left(\frac{2\pi}{3}\right)=-\cos\left(\frac{\pi}{3}\right)=\frac{-1}{2}$

$y=\sin\left(\frac{-44\pi}{3}\right)=-\sin\left(\frac{44\pi}{3}\right)$ odd function

$=-\sin\left(\frac{2\pi}{3}\right)=-\sin\left(\frac{\pi}{3}\right)=\frac{-\sqrt{3}}{2}$

Q2. Find the **range** and **phase shift** of the function $f(x)=-\sin(3x)-\cos(3x)-4$

The function $f(x)=-\sin(3x)-\cos(3x)-4=k\sin(3x+\alpha)-4$

$a=-1,\ b=-1\Rightarrow k=\sqrt{a^2+b^2}=\sqrt{(-1)^2+(-1)^2}=\sqrt{2}$

$\cos\alpha=\frac{a}{\sqrt{a^2+b^2}}=\frac{-1}{\sqrt{2}},\ \sin\alpha=\frac{b}{\sqrt{a^2+b^2}}=\frac{-1}{\sqrt{2}}\Rightarrow \alpha$ *lies in Q III*

$\alpha=\pi+\frac{\pi}{4}=\frac{5\pi}{4}$, then $f(x)=\sqrt{2}\sin\left(3x+\frac{5\pi}{4}\right)-4$

the range $\left[-\sqrt{2}-4,\sqrt{2}-4\right]$ and the phase shift $=\frac{-c}{b}=-\frac{-5\pi/4}{3}=\frac{-5\pi}{12}$

Q3. Find vertical asymptotes of the graph of the function $f(x)=-3\tan\left(\frac{3x}{2}+\frac{\pi}{3}\right)$

over the interval $[-2\pi,2\pi]$

$f(x)$ has V.A at $\frac{3x}{2}+\frac{\pi}{3}=\frac{\pi}{2}+k\pi$ k any integer, (solve for x)

$\frac{3x}{2}+\frac{\pi}{3}=\frac{\pi}{2}+k\pi\Rightarrow x=\frac{\pi}{9}+k\left(\frac{2\pi}{3}\right)$

when $k=0:\ x=\frac{\pi}{9},\ k=1:\ x=\frac{\pi}{9}+\frac{2\pi}{3}=\frac{7\pi}{9},\ k=2:\ x=\frac{\pi}{9}+2\left(\frac{2\pi}{3}\right)=\frac{13\pi}{9}$

$k=-1:\ x=\frac{\pi}{9}-\frac{2\pi}{3}=\frac{-5\pi}{9},\ k=-2:\ x=\frac{\pi}{9}-2\left(\frac{2\pi}{3}\right)=\frac{-11\pi}{9},\ k=-3:\ x=\frac{\pi}{9}-3\left(\frac{2\pi}{3}\right)=\frac{-17\pi}{9}$

V.A at $\left\{\frac{\pi}{9},\frac{7\pi}{9},\frac{13\pi}{9},\frac{-5\pi}{9},\frac{-11\pi}{9},\frac{-17\pi}{9}\right\}$

Q4. Verify each identity.

a) $\dfrac{\sin^3 x+\cos^3 x}{\sin x+\cos x}=1-\dfrac{1}{2}\sin 2x$

$$L.H.S=\frac{\sin^3 x+\cos^3 x}{\sin x+\cos x}=\frac{(\sin x+\cos x)\left(\sin^2 x-\sin x\cos x+\cos^2 x\right)}{\sin x+\cos x}=\sin^2 x-\sin x\cos x+\cos^2 x$$

$$=1-\sin x\cos x=1-\frac{1}{2}(2\sin x\cos x)=1-\frac{1}{2}\sin 2x=R.H.S$$

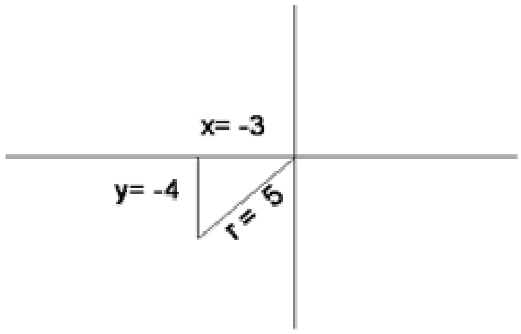
b) $2\tan\dfrac{x}{2}=\dfrac{\sin^2 x+1-\cos^2 x}{\sin x(1+\cos x)}$

$$R.H.S=\frac{\sin^2 x+1-\cos^2 x}{\sin x(1+\cos x)}=\frac{\sin^2 x+\sin^2 x}{\sin x(1+\cos x)}=\frac{2\sin^2 x}{\sin x(1+\cos x)}=\frac{2\sin x}{(1+\cos x)}=2\tan\frac{x}{2}=L.H.S$$

Q5. If $\tan\theta=\dfrac{4}{3}$ and θ is in Quadrant III. Find $\cos\left(\dfrac{\theta}{2}\right)+\cot(2\theta)$

θ *in Q* III, $x<0$ *and* $y<0$

$x=-3,\ y=-4 \Rightarrow r=\sqrt{16+9}=\sqrt{25}=5$



$$\cos\frac{\theta}{2}+\tan 2\theta=-\sqrt{\frac{1+\cos\theta}{2}}+\frac{1}{\tan 2\theta}=-\sqrt{\frac{1+\cos\theta}{2}}+\frac{1-\tan^2\theta}{2\tan\theta}$$

$$=-\sqrt{\frac{1+(-3/5)}{2}}+\frac{1-(4/3)^2}{2(4/3)}=-\sqrt{\frac{1}{5}}+\frac{1-16/9}{8/3}$$

$$=-\sqrt{\frac{1}{5}}+\frac{-7}{24}$$

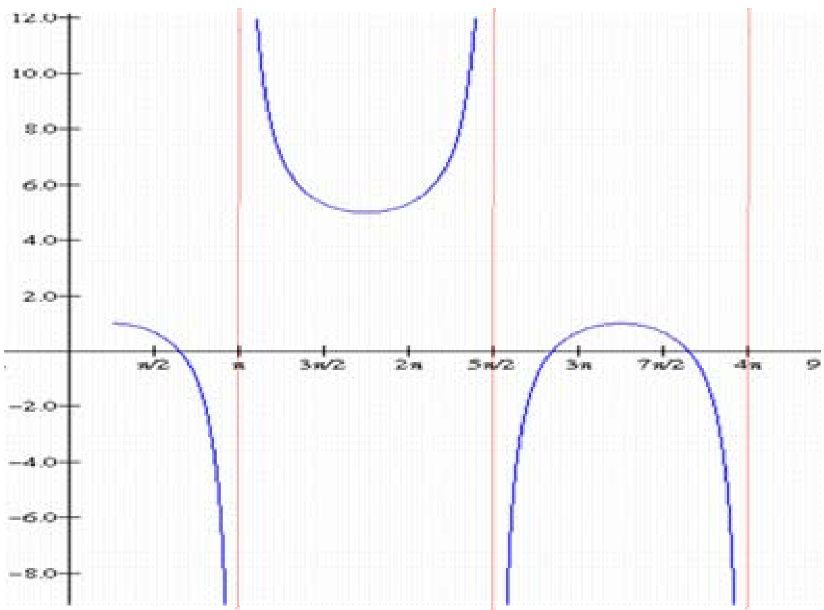
Q6 Graph $y=-2\sec\left(\dfrac{2x}{3}-\dfrac{\pi}{6}\right)+3$ over the interval $\left[\dfrac{\pi}{4},4\pi\right]$

Let $y=-2\cos\left(\dfrac{2x}{3}-\dfrac{\pi}{6}\right)+3$

Period $=\dfrac{2\pi}{|b|}=\dfrac{2\pi}{2/3}=3\pi$

Phase shift $=\dfrac{-c}{b}=\dfrac{-\pi/6}{2/3}$

$=\dfrac{\pi}{4}$ units to the right



	0	$\frac{3\pi}{4}$	$\frac{3\pi}{2}$	$\frac{9\pi}{4}$	3π	$\frac{15\pi}{4}$
x	$\frac{\pi}{4}$	π	$\frac{7\pi}{4}$	$\frac{10\pi}{4}$	$\frac{13\pi}{4}$	4π
y	1	V.A	5	V.A	1	V.A

Q7. Given the graph of $y = a\sin(bx + c) + d$, then find a+b+c+d

$$The\ period = start - end = \frac{5\pi}{4} - \frac{\pi}{4} = \pi$$

$$\frac{2\pi}{b} = \pi \Rightarrow b = 2$$

$$The\ phase\ shift = \frac{\pi}{4} = \frac{-c}{b} \Rightarrow \frac{\pi}{4} = \frac{-c}{2} \Rightarrow c = \frac{-\pi}{2}$$

$$a = \frac{\max - \min}{2} = \frac{4-0}{2} = 2$$

shift 2 units upward $d = 2$

$$a + b + c + d = 2 + 2 - \frac{\pi}{2} + 2 = 6 - \frac{\pi}{2}$$