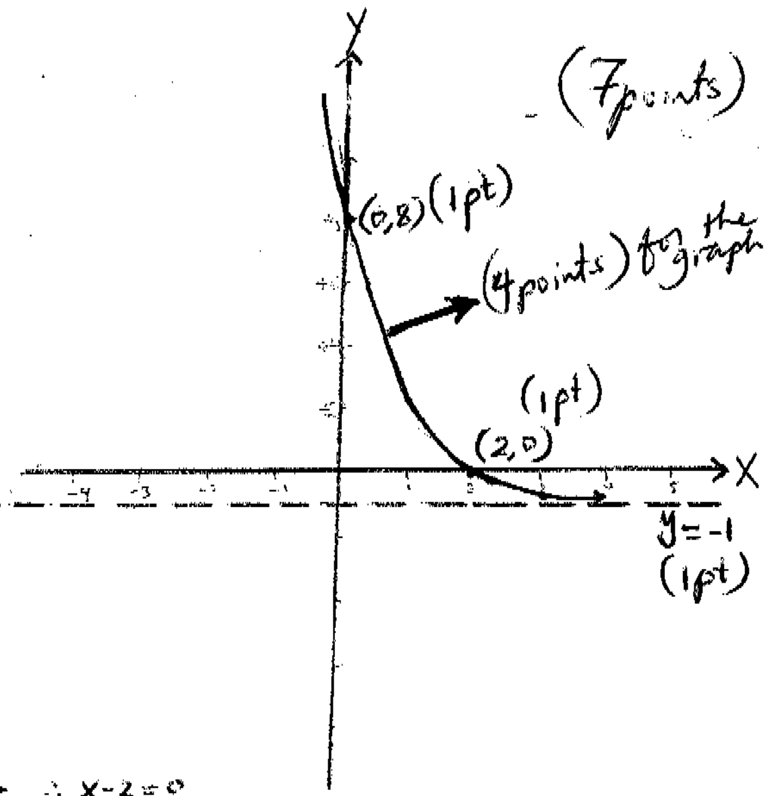
Q_1

$$f(x) = \left(\tfrac{1}{3}\right)^{x-2} - 1$$

(7 points)

x	0	2	3
y	8	0	$-\frac{2}{3}$



OR

Graph $f(x) = \left(\frac{1}{3}\right)^x$

as decreasing exp

then

a) shift right 2 units

b) shift down 1 unit

x-int $\Rightarrow$ let $y=0 \Rightarrow \left(\frac{1}{3}\right)^{x-2} = 1 \Rightarrow \therefore x-2=0$ $\boxed{x=2} \Rightarrow (2,0)$

y-int $\Rightarrow$ let $x=0 \Rightarrow y = \left(\frac{1}{3}\right)^{-2} - 1 = 9-1 = 8 \Rightarrow (0,8)$.

Q_2 IF $\log_2 5 = x$ and $\log_2 3 = y$, find $\log_{\sqrt{2}} 300$ (6 points)

$$\log_{\sqrt{2}} 300 = \frac{\log_2 300}{\log_2 \sqrt{2}} = \frac{\log_2 300}{\log_2 2^{\frac{1}{2}}} = 2\log_2 300 \quad (2pts)$$

$$= 2\left[\log_2 3 + \log_2 100\right]$$

$$= 2\left[\log_2 3 + \log_2 10^2\right]$$

$$= 2\left[\log_2 3 + 2\log_2 10\right] \quad (2pts)$$

$$= 2\left[\log_2 3 + 2\log_2 5 + 2\log_2 2\right] \quad (1pt)$$

$$= 2\left[y + 2x + 2\right]$$

$$= 4x + 2y + 4 \quad (1pt)$$

Q3

(6 points)

$$\frac{\sin t}{1+\cos t} + \frac{1+\cos t}{\sin t} = \frac{\sin^2 t + (1+\cos t)^2}{(\sin t)(1+\cos t)} \quad (2\text{pts})$$

$$= \frac{\sin^2 t + 1 + 2\cos t + \cos^2 t}{(\sin t)(1+\cos t)} \quad (1\text{pt.})$$

$$= \frac{2 + 2\cos t}{(\sin t)(1+\cos t)} \quad (1\text{pt.})$$

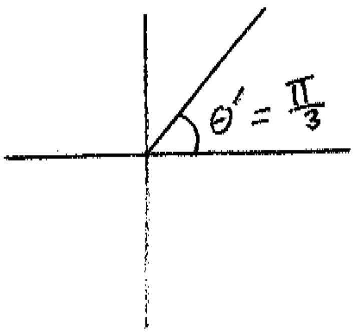
$$= \frac{2(1+\cos t)}{(\sin t)(1+\cos t)} = \frac{2}{\sin t} = 2\csc t$$

(2pts)

Q4 Find the values of x and y such that $w(-\frac{7\pi}{3}) = (x, y)$ (6pts)

$$X = \cos\left(-\frac{7\pi}{3}\right) = \cos\left(\frac{7\pi}{3}\right) = \cos\frac{\pi}{3} = \frac{1}{2} \quad (3\text{pts})$$

$$y = \sin\left(-\frac{7\pi}{3}\right) = -\sin\left(\frac{7\pi}{3}\right) = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2} \quad (3\text{pts})$$



Q_5 $\quad \log_3(-x) + \log_3(6-x) = 3$ (6 points)

$\Rightarrow \quad \log_3[(-x)(6-x)] = 3$ (2 pts)

$\Rightarrow \quad (-x)(6-x) = 3^3$ (1 pt.)

$\Rightarrow \quad x^2 - 6x - 27 = 0$

$\Rightarrow \quad (x-9)(x+3) = 0$ (1 pt.)

$\Rightarrow \quad x-9=0 \quad$ or $\quad x+3=0$

$\Rightarrow \quad x = 9 \quad$ or $\quad x = -3$

Check! (1 pt.) will give the Solution set $= \{-3\}$ (1 pt.)

Q_6 a) Find the exact value of $\sin^2 45^\circ + \cos^2 60^\circ$ (3 poi…

$$\sin^2 45^0 + \cos^2 60^0 = \underset{(1\text{pt.})}{\left(\frac{1}{\sqrt{2}}\right)^2} + \underset{(1\text{pt.})}{\left(\frac{1}{2}\right)^2} = \frac{1}{2} + \frac{1}{4}$$

$$= \frac{3}{4} \text{ (1 pt)}$$

b) A wheel is rotating at 100 revolutions per minute. Find the angular speed in radian per seconds. (5 points)

$$100 \text{ rev}/\text{minute} = \left(\frac{100 \text{ rev}}{1 \text{ minute}}\right)\underset{(2\text{pts})}{\left(\frac{2\pi \text{ radians}}{1 \text{ rev}}\right)}\underset{(1\text{pt.})}{\left(\frac{1 \text{ minute}}{60 \text{ seconds}}\right)}$$

$$= \underset{(1\text{pt.})}{\frac{10\pi}{3}} \quad \underset{(1\text{pt})}{\text{rad}/s}$$

Q7 $P(t) = 16000\, e^{2t}$. Find the time when the population reaches 48000. (5 points)

$48000 = 16000\, e^{2t} \Rightarrow e^{2t} = 3 \Rightarrow 2t = \ln 3$ (pt.)

(2 pts)

$\Rightarrow t = \frac{1}{2}\ln 3$

$\Rightarrow t = \ln\sqrt{3}$ years.

(1 pt.) (1 pt)

Q8 $f(x) = -\frac{1}{2} + \operatorname{Log}_9(1-2x)$ (9 points)

a) Domain: $1-2x > 0$

$-2x > -1$

$x < \frac{1}{2}$ or $D = (-\infty, \frac{1}{2})$

b) Range: $R = (-\infty, \infty)$

c) Vertical Asymptote: $1-2x = 0 \Rightarrow x = \frac{1}{2}$

d) X-int $\Rightarrow y = 0 \Rightarrow -\frac{1}{2} + \operatorname{Log}_9(1-2x) = 0$

$\Rightarrow \operatorname{Log}_9(1-2x) = \frac{1}{2}$

$\Rightarrow 1-2x = 3 \Rightarrow -2x = 2 \Rightarrow x = -1$

$\Rightarrow (-1, 0)$

e) Y-int $\Rightarrow x = 0 \Rightarrow y = -\frac{1}{2} + \operatorname{Log}_9 1 = -\frac{1}{2}$

$\Rightarrow (0, -\frac{1}{2})$

Q.8
(a),(b),(c) $f(x) = -\frac{1}{2} + \log_9 (1-2x)$ ((5 points))

a) Domain, $1-2x > 0$ (1pt)
$-2x > -1$
$x < \frac{1}{2} \Rightarrow D: (-\infty, \frac{1}{2})$ (1pt.)

b) Range: $\mathbb{R}$ or $(-\infty, \infty)$ or All real numbers
((1 point))

c) Vertical Asymptote: $1-2x=0$ (1pt.) $\Rightarrow \boxed{x=\frac{1}{2}}$ (1pt.)

Q.8
(d),(e) $f(x) = -\frac{1}{2} + \log_9 (1-2x)$ ((4 points))

d) X-int, let $y=0 \Rightarrow -\frac{1}{2} + \log_9(1-2x) = 0$
$\Rightarrow \log_9(1-2x) = \frac{1}{2} \Rightarrow$ (2pts)
$1-2x = 9^{1/2}$
$1-2x = 3$
$x = -1$
$\Rightarrow$ X-int $(-1, 0)$

e) y-int, let $x=0 \Rightarrow y = -\frac{1}{2} + \log_9 1 = -\frac{1}{2} + 0$
$= -\frac{1}{2}$ (2pts)
$\Rightarrow$ y-int $(0, -\frac{1}{2})$

Q9 $f(x) = a\cos bx$, $b > 0$, Period $= 6$ and $f(3) = 4$

Find $f(\frac{21}{4})$. (7 points)

Period $= \frac{2\pi}{b} = 6 \Rightarrow b = \frac{2\pi}{6} = \frac{\pi}{3}$ (2 pts)

also, $f(3) = a\cos(\frac{\pi}{3})(3) = a\cos\pi = -a = 4$

$\therefore a = -4$ (2 pts)

So, $f(x) = -4\cos(\frac{\pi}{3}x)$ (1 pt.)

and $f(\frac{21}{4}) = -4\cos(\frac{\pi}{3}\cdot\frac{21}{4}) = -4\cos\frac{7\pi}{4}$

$= -4\cos\frac{\pi}{4}$

$= (-4)(\frac{\sqrt{2}}{2}) = -2\sqrt{2}$ (2 pts)

Q10

a) $d = 14$ cm $\Rightarrow r = 7$ cm (1 pt) (5 points)

$\theta = 150° = (150°)\left(\frac{\pi \text{ radians}}{180°}\right) = \frac{5\pi}{6}$ radians (1 pt.)

$\underbrace{S = r\theta}_{(1\text{ pt.})} = (7)\left(\frac{5\pi}{6}\right) = \underbrace{\frac{35\pi}{6}}_{(1\text{ pt})}\ \underbrace{\text{cm.}}_{(1\text{ pt})}$

b) Find x if $5\log_2(\log_4 16) + x = 1 + 2\ln e^x$ (5 points)

$5\log_2(\log_4 4^2) + x = 1 + 2x$ (3 pts)

$5\log_2 2 + x = 1 + 2x$

$5 + x = 1 + 2x$ (1 pt)

$\boxed{x = 4}$ (1 pt.)

Q11

a) $f(x) = x - \sin x$ (4 points)

$f(-x) = -x - \sin(-x) = -x + \sin x$ (2pts)

$= -(x - \sin x) = -f(x)$ (1pt) $\therefore f$ is an odd function (1pt)

b) $6\frac{1}{3}$ terminates in the First quadrant (4 points)

$2\pi \approx 6.28 < 6\frac{1}{3} < 7.85 \approx \frac{5\pi}{2}$

Q12 $y = -\left|3 \sin \frac{2x}{3}\right|$ (4 points)

Graph first $y = 3\sin\frac{2}{3}x$

Amp $= |3| = 3$

period $= \frac{2\pi}{|b|} = \frac{2\pi}{\frac{2}{3}} = 3\pi$

for $y = -\left|3\sin\frac{2}{3}x\right|$

- values of y will be $\ominus$
- period $= \frac{3\pi}{2}$

Period	max	min
$\frac{3\pi}{2}$	0	-3
(1pt)	(1pt)	(1pt)