

Q1. . Solve $2\log_5(1-x) + \log_{\frac{1}{5}}(x-2) = 2\log_5 2$.

$$\log_5(1-x)^2 + \frac{\log_5(x-2)}{\log_5\left(\frac{1}{5}\right)} = \log_5 2^2 \rightarrow \log_5(1-x)^2 + \frac{\log_5(x-2)}{\log_5 5^{-1}} = \log_5 4$$

$$\log_5(1-x)^2 - \log_5(x-2) = \log_5 4 \rightarrow \log_5 \frac{(1-x)^2}{(x-2)} = \log_5 4$$

$$\frac{(1-x)^2}{(x-2)} = 4 \rightarrow \frac{1-2x+x^2}{(x-2)} = 4 \rightarrow 1-2x+x^2 = 4x-8$$

$$x^2 - 6x + 9 = 0 \rightarrow (x-3)(x-3) = 0 \rightarrow x = 3$$

check $x=3$ reject, is not in domain of $\log_5(1-x)$

So no solution.

Q2. If the amount of a certain radioactive material present after t days is $P(t) = 800e^{-t \ln 2}$ grams, find the time needed for the material to decay to 400 grams.

$$400 = 800e^{-t \ln 2} \rightarrow \frac{400}{800} = e^{-t \ln 2} \rightarrow \frac{1}{2} = e^{-t \ln 2} \rightarrow 2^{-1} = e^{\ln 2^{-t}}$$

$$2^{-1} = 2^{-t} \rightarrow -1 = -t \rightarrow t = 1 \text{ day.}$$

Q3. Solve for x $(16)^4 = 2(8)^{|x+1|}$

$$(2^4)^4 = 2(2^3)^{|x+1|} \rightarrow 2^{16} = 2(2)^{3|x+1|} \rightarrow 2^{16} = 2^{1+3|x+1|} \rightarrow 16 = 1+3|x+1|$$

$$15 = 3|x+1| \rightarrow 5 = |x+1| \rightarrow x+1 = \pm 5 \rightarrow x = 4, x = -6$$

Q4. Evaluate $(\log_5 16)(\log_2 \sqrt{5}) - (\sqrt{e})^{-6 \ln 2}$

$$= \left(\frac{\log_2 2^4}{\log_2 5} \right) \log_2 (5)^{\frac{1}{2}} - \left(e^{\frac{1}{2}} \right)^{-6 \ln 2} = \left(\frac{4}{\log_2 5} \right) \left(\frac{1}{2} \log_2 (5) \right) - (e)^{-3 \ln 2}$$

$$= 4 \left(\frac{1}{2} \right) - e^{\ln 2^{-3}} = 2 - 2^{-3} = 2 - \frac{1}{8} = \frac{15}{8}$$