QUIZ # 7&8(2) **MATH002** **Sec.# 22&12** **Sr.#** **Name:**

SHOW ALL YOUR WORK

Q1. The graph of $4x^2 - 9y^2 - 16x + 54y - 29 = 0$ **is:**

a) **hyperbola with vertices (5,2) and (1,2)**
b) **ellipse with vertices (2,5) and (2,1)**
c) **hyperbola with vertices (2,5) and (2,1)**
d) **hyperbola with vertices (2,2) and (1,2)**
e) **ellipse with vertices (5,2) and (1,2)**

Solution

$$4\left[x^2-4x\right]-9\left[y^2-6y\right]=29 \rightarrow 4\left[x^2-4x+2^2-2^2\right]-9\left[y^2-6y+3^2-3^2\right]=29$$

$$\rightarrow 4\left[(x-2)^2-4\right]-9\left[(y-3)^2-9\right]=29 \rightarrow 4(x-2)^2-16-9(y-3)^2+81=29$$

$$\rightarrow 4(x-2)^2-9(y-3)^2=29+16-81 \rightarrow 4(x-2)^2-9(y-3)^2=-36 \rightarrow \frac{(y-3)^2}{4}-\frac{(x-2)^2}{9}=1$$

Vertical hyperbola center $(2,3)$ $a=2, b=3$ *vertices* $(2,5)\&(2,1)$

Q2. If (a,b) and (c,d) **are solutions of the following system**

$$2x^2-4xy-y^2=6 \qquad (1)$$

$$4x^2-3xy-y^2=6. \qquad (2)$$

Then $abcd=$

a) -4 b) 6 c) 2 d) – 6 e) 4

Solution

eq.(1) - eq.(2) $\rightarrow$

$$\begin{array}{l} 2x^2-4xy-y^2=6 \\ \underline{-4x^2+3xy+y^2=-6.} \\ -2x^2-xy+0=0 \end{array}$$

$$2x^2+xy=0 \rightarrow x(2x+y)=0 \rightarrow x=0 \text{ or } x=-\frac{1}{2}y$$

Substitute $x=0$ *in eq.*$(1) \rightarrow 0+0-y^2=6 \rightarrow y^2=-6 \rightarrow$ *nonreal solution*

Substitute $x=-\frac{1}{2}y$ *in eq.*$(1) \rightarrow 2\frac{y^2}{4}-4\frac{y}{2}y+y^2=6 \rightarrow \frac{3}{2}y^2=6 \rightarrow y^2=4 \rightarrow y=\pm 2$

Substitute $y=\pm 2$ *in* $x=-\frac{1}{2}y$

$y=2 \rightarrow x=-1 \rightarrow (-1,2),\ y=-2 \rightarrow x=1 \rightarrow (1,-2)$

then $-1(2)(1)(-2)=4$

Continue

Q3. **The equation of the ellipse having center (3 , -2), vertex (3 , 3), and focus (3 , 1) is:**

a) $\frac{(x-3)^2}{25}+\frac{(y+2)^2}{16}=1$

b) $\frac{(x-3)^2}{16}+\frac{(y+2)^2}{25}=1$

c) $\frac{(x+3)^2}{16}+\frac{(y-2)^2}{125}=1$

d) $\frac{(x-3)^2}{16}+\frac{(y-2)^2}{25}=1$

e) $\frac{(x-2)^2}{25}+\frac{(y+3)^2}{16}=1$

Solution

Since center, vertex, & focus lie on vertical line → vertical – ellipse

a = distance between center & vertex = 5

b = distance between center & focus = 3

$c^2=a^2-b^2 \to 9=25-b^2 \to b^2=16$

Q4. **Given the vectors** $\vec{u}=\langle 1,2\rangle$ and $\vec{v}=\langle -3,\sqrt{3}-1\rangle$ **then the magnitude M and direction angle of the vector** $\vec{u}+\vec{v}+\vec{i}-\vec{j}$ **are given by:**

(a) $M=4$ **and** $\theta=135^\circ$

(b) $M=2$ *and* $\theta=150^\circ$

(c) $M=2$ *and* $\theta=135^\circ$

(d) $M=2$ *and* $\theta=120^\circ$

(e) $M=4$ *and* $\theta=120^\circ$

Solution

$\vec{u}+\vec{v}+\vec{i}-\vec{j}=i+2j-3i+(\sqrt{3}-1)j+i-j=-i+\sqrt{3}j$

$M=\sqrt{(1)^2+(\sqrt{3})^2}=2$

$\tan\theta=\frac{\sqrt{3}}{-1}=-\sqrt{3} \to \theta'=60^\circ$ *Since* $-i+\sqrt{3}j$ *in Quadrant II*

$\theta=120^\circ$

Continue

Q5.If the following system is dependent. Then the values of <u>a & b</u> are equal:

$$bx + 2y = 4$$
$$2x - y = 2a.$$

a) $b = -4$ *and* $a = -1$
b) $b = 4$ *and* $a = 1$
c) $b = -4$ *and* $a = 1$
d) $b = 4$ *and* $a = -1$
e) $b = -4$ *and* $a = -2$

Solution

$eq.(1) + 2 \times eq.(2) \rightarrow$

$bx + 2y = 4$

$4x - 2y = 4a.$

$bx + 4x + 0 = 4 + 4a$

$(b+4)x = 4 + 4a$

Since the system is dependent $\rightarrow b + 4 = 0 \ \& \ 4 + 4a = 0$

$\rightarrow b = -4 \ \& \ a = -1$

Q6. The equation of the parabola with directrix $x = -1$ and focus $(3, 2)$ is:

(a) $(y-2)^2 = 4(x-1)$

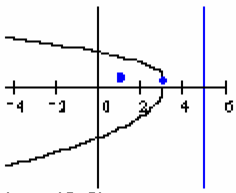
(b) $(y-2)^2 = -8(x-3)$

(c) $(y+2)^2 = 8(x+3).$

(d) $(x-1)^2 = -8(y-2).$

(e) $(x-2)^2 = 4(y-1)$

Solution



vertex = midpoint between focus & directrix = (3, 2),

Since the vertex & the focus lie on horizontal line then the axis is horizontal

$(y-k)^2 = 4p(x-h)$

parabola open to the left $p = -(\text{distance between vertex \& focus}) = -2$