K.F.U.P.M.

QUIZ # 6 MATH002 Sec.# Sr.# Name:

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Q1. If $6\cos^2 15^o - 3 =$

(a) $\frac{3\sqrt{3}}{2}$

(b) $\sqrt{3}$

(c) $\frac{3}{2}$

(d) $\frac{\sqrt{3}}{2}$

$$6\cos^2 15^o - 3 = 3(2\cos^2 15^o - 1) = 3\cos(2\times 15) = 3\cos 30^o = 3\frac{\sqrt{3}}{2}$$

Q3. If $f(x) = -\sin 2x + \sqrt{3}\cos 2x$. Write $f(x) = k\sin(bx + \alpha)$, then $\alpha =$

(a) $\frac{\pi}{6}$

(b) $\frac{2\pi}{3}$

(c) π

(d) $\frac{\pi}{3}$

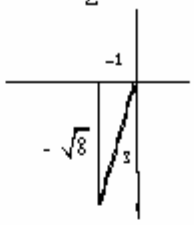
$$k = \sqrt{(-1)^2 + \left(\sqrt{3}\right)^2} = \sqrt{4} = 2$$

$$\cos(\alpha) = \frac{-1}{2},\ \sin(\alpha) = \frac{\sqrt{3}}{2} \rightarrow \alpha \text{ in Quadrant II}$$

$$\alpha' = \frac{\pi}{3},\ \rightarrow \frac{\pi}{3} = \pi - \alpha,\ \rightarrow \alpha = \frac{2\pi}{3}$$

Q2. If $\sec\theta = -3$, where θ in the third quadrant,then find: .

a) $\csc 2\theta$ b) $\sin\frac{\theta}{2}$



a) $\csc(2\Theta) = \frac{1}{\sin(2\Theta)}$, $\sin(2\Theta) = 2\sin\Theta\cos\Theta$

$\sin(2\Theta) = 2\left(\frac{-\sqrt{8}}{3}\right)\left(\frac{-1}{3}\right) = \frac{2\sqrt{8}}{9}$

Then $\csc 2\Theta = \frac{9}{2\sqrt{8}} = \frac{9\sqrt{8}}{16} = \frac{9\times 2\sqrt{2}}{16} = \frac{9\sqrt{2}}{8}$

b)

Θ in Quadrant III, $\rightarrow \pi < \Theta < \frac{3\pi}{2}, \rightarrow \frac{\pi}{2} < \frac{\Theta}{2} < \frac{3\pi}{4}, \rightarrow \frac{\Theta}{2}$ in Quadrant II

$\sin\frac{\Theta}{2} = \pm\sqrt{\frac{1-\cos\Theta}{2}}, \rightarrow \sin\frac{\Theta}{2} = \sqrt{\frac{1-\cos\Theta}{2}} = \sqrt{\frac{1-\frac{-1}{3}}{2}} = \sqrt{\frac{\frac{4}{3}}{2}} = \sqrt{\frac{2}{3}} = \frac{\sqrt{6}}{3}$

Q3. Verify

a) $\frac{\sin^3 x + \cos^3 x}{\sin x + \cos x} = 1 - \frac{1}{2}\sin 2x$ b) $\frac{1}{2}(\tan x + \cot x) = \csc 2x$

a)

$$\frac{\sin^3 x + \cos^3 x}{\sin x + \cos x} = \frac{(\sin x + \cos x)(\sin^2 x - \sin x\cos x + \cos^2 x)}{\sin x + \cos x}$$

$$= (\sin^2 x - \sin x\cos x + \cos^2 x)$$

$$= 1 - \sin x\cos x = 1 - \frac{1}{2}(2\sin x\cos x) = 1 - \frac{1}{2}\sin 2x = \text{R.H.S.}$$

b)

$$\frac{1}{2}(\tan x + \cot x) = \frac{1}{2}\left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right) = \frac{1}{2}\left(\frac{\sin^2 x + \cos^2 x}{\sin x\cos x}\right) = \frac{1}{2}\left(\frac{1}{\sin x\cos x}\right)$$

$$= \frac{1}{2\sin x\cos x} = \frac{1}{\sin 2x} = \csc 2x$$