

1] If $f(x) = 2\sec\left(2x + \frac{\pi}{3}\right) - 1$

a) Find the amplitude of f if any.

No amplitude

b) Find the period of f

$$P = \frac{2\pi}{b} = \frac{2\pi}{2} = \pi$$

c) Find the phase shift of f

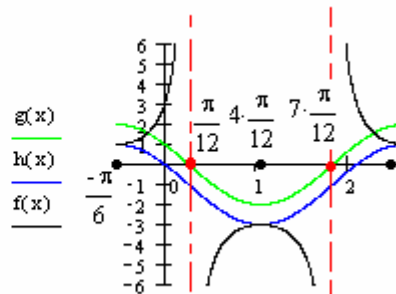
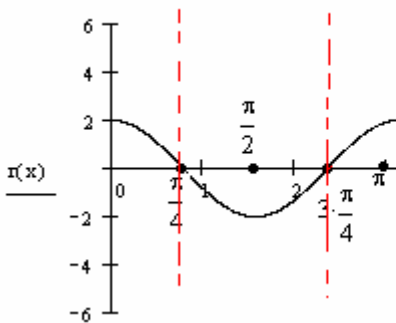
$$-\frac{c}{b} = -\frac{\frac{\pi}{3}}{2} = -\frac{\pi}{6} \text{ to the left.}$$

d) Find the equation of vertical asymptote

$$x = \frac{\pi}{12} + \frac{n}{2}(\pi) = \left(\frac{1+6n}{12}\right)\pi, \text{ where } n \text{ is any integer}$$

e) Draw the graph of the function f over one period

$$r(x) = 2\cos(2x), \quad g(x) = 2\cos\left(2x + \frac{\pi}{3}\right), \quad h(x) = 2\cos\left(2x + \frac{\pi}{3}\right) - 1, \quad f(x) = 2\sec\left(2x + \frac{\pi}{3}\right) - 1$$

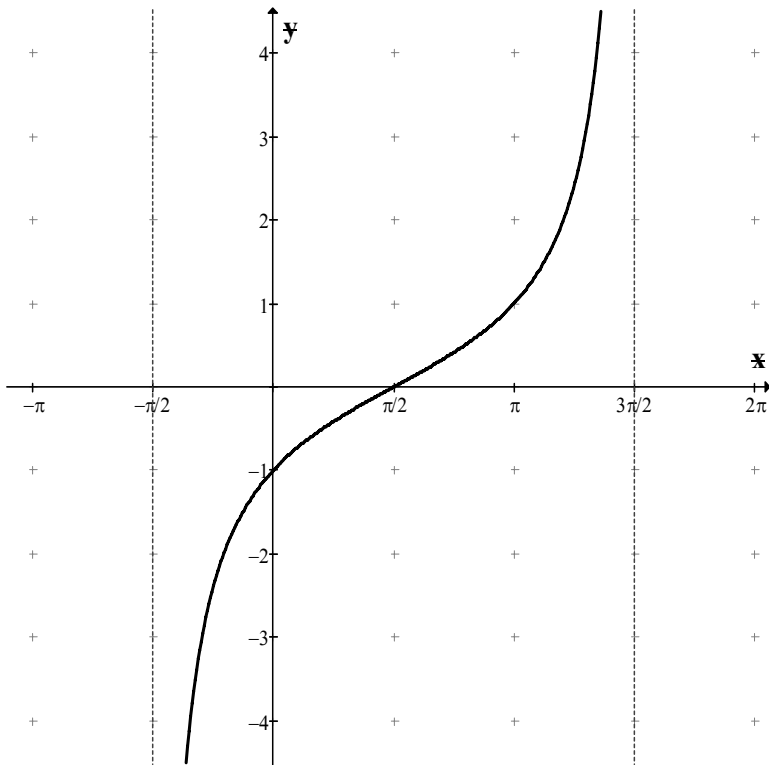


2] Find the period and the amplitude of $f(x) = -\left|\sin\left(2x + \frac{\pi}{6}\right)\right|$

The period of $y = \sin\left(2x + \frac{\pi}{2}\right)$ is $\frac{2\pi}{2} = \pi$, then the period of $\left|\sin\left(2x + \frac{\pi}{2}\right)\right|$ is $\frac{\pi}{2}$

The amplitude of $y = \sin\left(2x + \frac{\pi}{2}\right)$ is 1, then the amplitude of $\left|\sin\left(2x + \frac{\pi}{2}\right)\right|$ is $\frac{1}{2}$

5] Find an equation of the form $f(x) = a \tan(bx + c)$ of the given graph



The period of $f(x)$ is $P = \frac{3\pi}{2} - \left(-\frac{\pi}{2}\right) = 2\pi = \frac{\pi}{b} \rightarrow b = \frac{1}{2}$

The phase shift is $-\frac{c}{b} = -\frac{c}{1/2} = \frac{\pi}{2} \rightarrow c = -\frac{\pi}{4}$

Then $f(x) = a \tan\left(\frac{x}{2} - \frac{\pi}{4}\right)$

From the graph $f(\pi) = 1 \rightarrow 1 = a \tan\left(\frac{\pi}{2} - \frac{\pi}{4}\right) = a \tan\left(\frac{\pi}{4}\right)$

Then $1 = a(1) \rightarrow a = 1$, then $f(x) = \tan\left(\frac{x}{2} - \frac{\pi}{4}\right)$