KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS
MATHEMATICAL DEPARTMENT

Math. 002 First Test Term(042)

Name: I.D.# Sec.#:

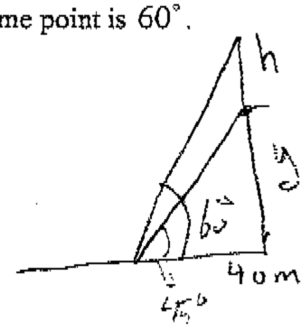
Q1. If $\log 2 = x$ *and* $\log 3 = y$, then write $\log_5 900 + \log(2+3)$

$\log_5 900 + \log(2+3) = \frac{\log 900}{\log 5} + \log 5 = \frac{\log 9 + \log 100}{\log \frac{10}{2}} + \log \frac{10}{2}$

$= \frac{2\log 3 + 2}{\log 10 - \log 2} + \log 10 - \log 2 = \frac{2y+2}{1-x} + 1 - x = \frac{2y+2+(1-x)^2}{1-x}$

5

Q2. The angle of elevation of the top of an unfinished tower at a point 40 meters from its base is 45°. How much higher must be the tower raused so that the angle of elevation at the same point is 60°.



$\tan 45^\circ = \frac{y}{40} \rightarrow 1 = \frac{y}{40} \rightarrow y = 40$

$\tan 60^\circ = \frac{h+y}{40} \rightarrow \sqrt{3} = \frac{h+40}{40}$

$\rightarrow \sqrt{3}(40) = h + 40 \rightarrow h = 40\sqrt{3} - 40$

$= 40(\sqrt{3}-1)\,m$

5

Q3. Find the inverse function of $f(x) = \ln(x+1) - 2$

$y = \ln(x+1) - 2$

$x = \ln(y+1) - 2$

$x + 2 = \ln(y+1)$

$e^{x+2} = y + 1$

$e^{x+2} - 1 = y$

$\therefore f^{-1}(x) = e^{x+2} - 1$

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Q4. Given the function $g(x) = -2^{2-x} + 8$. Find x-intercept, y-intercept, domain, range, and the asymptote of the graph of g. Sketch the graph of g.

8

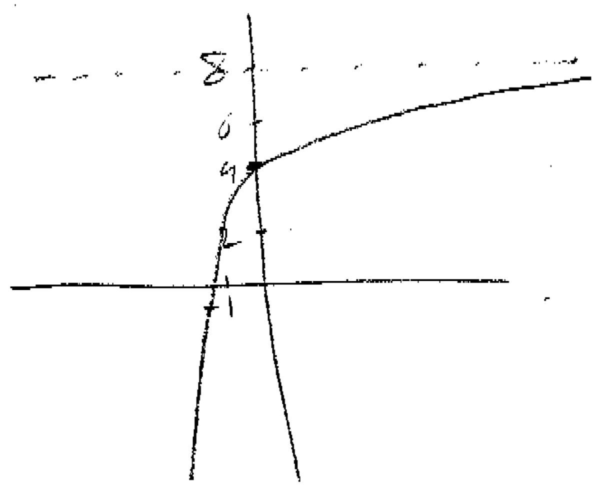
X-int $0 = -2^{2-x} + 8 \rightarrow 2^{2-x} = 2^3 \rightarrow 2 - x = 3 \rightarrow \underline{x = -1}$

Y-int $y = -2^{2-0} + 8 = -4 + 8 = 4 \rightarrow \underline{y = 4}$

domain $D = (-\infty, \infty)$

H.A. $y = 8$

Range $R = (-\infty, 8)$



Q5. If the terminal side of angle θ in standard position in second quadrant has equation $y = -2x$ then fin the value of $2\sec^2\theta - 3\cot\theta$,

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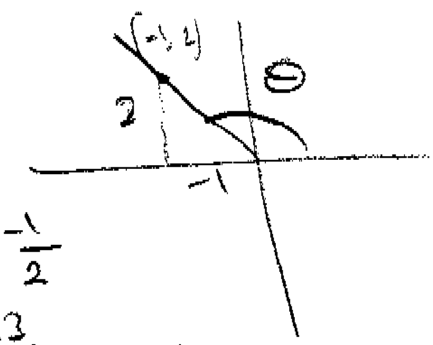
take $x = -1 \rightarrow y = 2 \Rightarrow (-1, 2)$

$r = \sqrt{1+4} = \sqrt{5}$

$\sec\theta = \frac{r}{x} = \frac{\sqrt{5}}{-1}$, $\cot\theta = \frac{x}{y} = \frac{-1}{2}$

$\Rightarrow 2(5) - 3\left(\frac{-1}{2}\right) = 10 + \frac{3}{2} = \frac{23}{2}$



Q6. Find the solution set of the equation
$\log(4-x) = \log(x+8) - \log_{10^{-1}}(2x+13)$

7

$\log(4-x) = \log(x+8) - \dfrac{\log(2x+13)}{\log 10^{-1}}$

$\log(4-x) = \log(x+8) + \log(2x+13)$

$\log(4-x) = \log(x+8)(2x+13)$

$4-x = (x+8)(2x+13) = 2x^2 + 29x + 104$

$4-x = 2x^2 + 29x + 104$

$\rightarrow 0 = 2x^2 + 30x + 100$

$x^2 + 15x + 50 = 0$

$(x+10)(x+5) = 0$

$x = -10 \quad x = -5$

Check $x = -10$ reject

$x = -5$

S.S. $= \{-5\}$

Q7. Find the length of arc of a circle with diameter 18 cm and central angle $\theta = 40^\circ$

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$r = 9\text{cm} \qquad \theta = \dfrac{40 \cdot \pi}{180}$ rad

$S = r\theta = \cancel{9} \cdot \dfrac{4\cancel{0}\pi}{\cancel{180}_{2}} = 2\pi\,\text{cm}$

Q8.a) Find $\sec(-210^\circ) + \cot\left(-\dfrac{35\pi}{4}\right)$ $= \sec 210^\circ - \cot\left(\dfrac{35\pi}{4}\right)$

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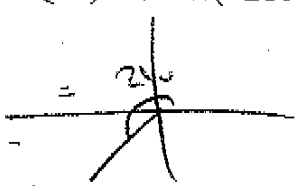
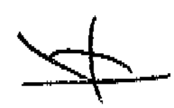


$= 210^\circ$

$\theta' = 30^\circ$

$\dfrac{35\pi}{4} = \dfrac{32\pi + 3\pi}{4} = 8\pi + \dfrac{3\pi}{4} \equiv \dfrac{3\pi}{4}$



$\theta' = \dfrac{\pi}{4}$

$\therefore \sec 210^\circ - \cot\left(\dfrac{35\pi}{4}\right)$

$= -\sec 30^\circ - \left[-\cot\dfrac{\pi}{4}\right] = -\dfrac{2}{\sqrt{3}} + 1 = \dfrac{-2+\sqrt{3}}{\sqrt{3}}$

b) Find the supplement angle of $130^\circ 25' 32''$

$179^\circ\ 59'\ 60''$
$130^\circ\ 25'\ 32''$
$49^\circ\ 34'\ 28''$

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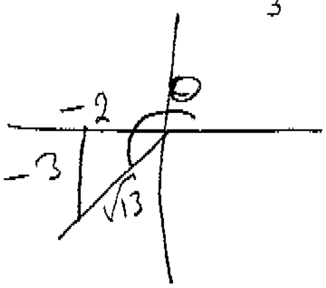
Q9. *If* $\cot\theta = \frac{2}{3}$ *and* $\sin\theta < 0$, *find* $\cos\theta - \csc\theta$



$\cos\theta = \frac{-2}{\sqrt{13}} - \frac{\sqrt{13}}{-3} = -\frac{2}{\sqrt{13}} + \frac{\sqrt{13}}{3}$

$= \frac{-6+13}{3\sqrt{13}} = \frac{7}{3\sqrt{13}} = \frac{7\sqrt{13}}{39}$

Q10. **Show all your work**

$\frac{1-\sin x}{\cos x} - \frac{1}{\tan x + \sec x} =$

a) 0
b) $\cos x$
c) $\sin x$
d) 1
e) $\tan x$

$\frac{1-\sin x}{\cos x} - \frac{1}{\frac{\sin x}{\cos x} + \frac{1}{\cos x}}$

$= \frac{1-\sin x}{\cos x} - \frac{\cos x}{\sin x + 1}$

$= \frac{1 - \sin^2 x - \cos^2 x}{(\cos x)(\sin x + 1)} = \frac{1 - (\sin^2 x + \cos^2 x)}{\cos x(\sin x + 1)}$

$= \frac{1-1}{\cos x(\sin x + 1)} = 0$