1. If $y' = \dfrac{1}{x\sqrt{1-(\ln x)^2}}$ and $y(1) = 5$, find y.

$$y = \int_1^x \frac{1}{t\sqrt{1-[\ln t]^2}}\,dt + 5, \quad \text{Let } u = \ln t \rightarrow du = \frac{1}{t}\,d$$

$$= \int_0^{\ln x} \frac{1}{\sqrt{1-u^2}}\,du + 5 = \sin^{-1} u \Big]_0^{\ln x} + 5$$

$$= \sin^{-1} \ln x - \sin^{-1} 0 + 5$$

$$= \sin^{-1} \ln x + 5$$

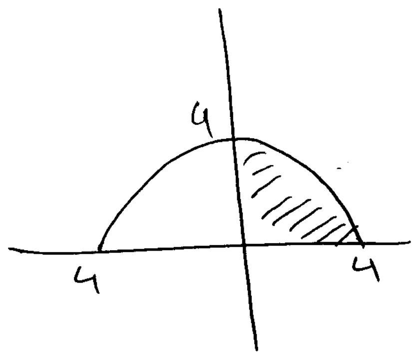
2. Express $\sum_{k=3}^{9} k e^{k-4}$ with $k = 1$ as the lower limit of summation.

$$= \sum_{k=1}^{7} (k+2)\, e^{k-2}$$

3. Evaluate $\int_0^{4/3} \sqrt{16 - 9x^2}\, dx$

Let $u = 3x \rightarrow du = 3dx$

$$I = \frac{1}{3}\int_0^4 \sqrt{16-u^2}\,du$$



$$= \frac{1}{3} \cdot \left(\frac{1}{4}\pi (4)^2\right) = \frac{4\pi}{3}$$

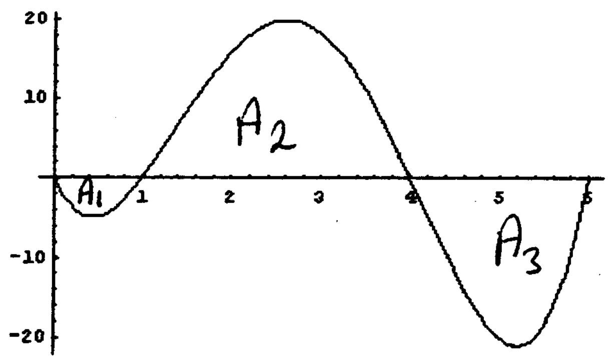
4. If $F(x) = \int_{\sqrt{3}}^{2x} \tan^{-1} z\,dz$, find

a. $F\left(\frac{\sqrt{3}}{2}\right)$ $= \int_{\sqrt{3}}^{\sqrt{3}} \tan^{-1} z\,dz = 0$

b. $F'\left(\frac{1}{2}\right)$, $F'(x) = (\tan^{-1} 2x)\,2 - 0$

$F'(1/2) = 2\tan^{-1} 1 = 2 \cdot \frac{\pi}{4} = \pi/2$

5. Consider the given graph of $f(x)$. If the area $A_1 = 4$, area $A_2 = 22$, and area $A_3 = 12$, find



a. $\int_0^1 f(x)\,dx$ $= -4$

b. $\int_1^6 f(x)\,dx$ $= A_2 - A_3 = 22 - 12 = 10$

c. the area bounded by $f(x)$ and the x-axis.

$= A_1 + A_2 + A_3 = 38$

6. Find the average value of $f(x)=\dfrac{e^{\sqrt{x+1}}}{\sqrt{x+1}}$ over $[0,3]$

$$f(x^*) = \frac{1}{3-0}\int_0^3 \frac{e^{\sqrt{x+1}}}{\sqrt{x+1}}\,dx$$

Let $u = \sqrt{x+1} \rightarrow du = \dfrac{1}{2\sqrt{x+1}}\,dx$

$$f(x^*) = \frac{1}{3}\, 2\int_1^2 e^u\,du = \frac{2}{3}\left[e^u\right]_1^2 = \frac{2}{3}\left[e^2 - e\right]$$

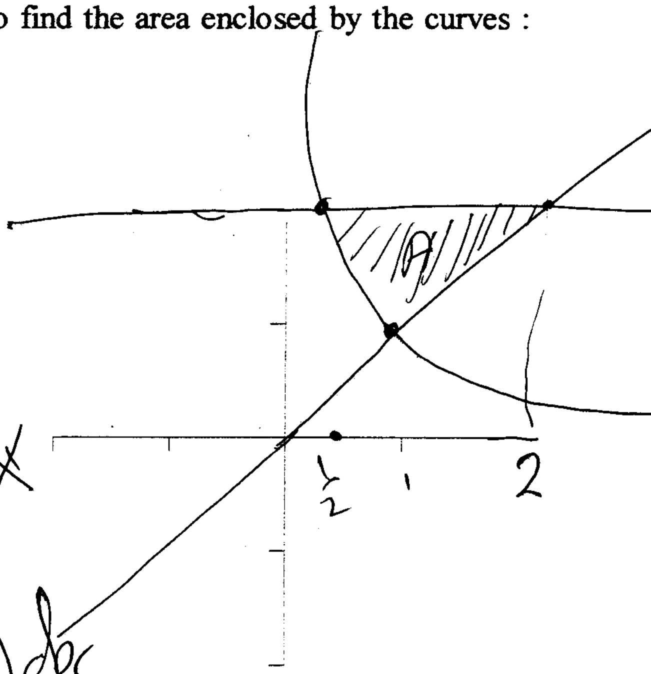
7. Set up (***Do NOT evaluate***) the integral needed to find the area enclosed by the curves :

$y = x$, $y = \dfrac{1}{x}$ and $y = 2$

$y = \frac{1}{x}$ and $y = 2 \Rightarrow \frac{1}{x} = 2 \rightarrow x = \frac{1}{2}$

$y = x, y = 2 \rightarrow x = 2$

$y = x$ and $y = \frac{1}{x} \rightarrow x = \frac{1}{x} \rightarrow x = 1, \cancel{-1}$



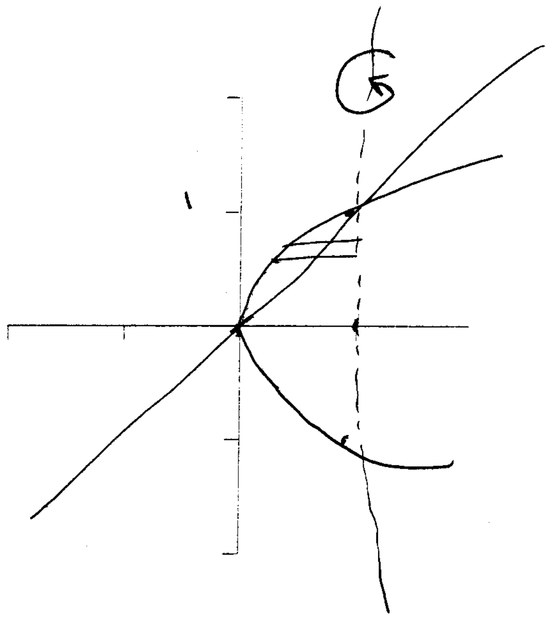
$$A = \int_{1/2}^{1}\left(2 - \frac{1}{x}\right)dx + \int_1^2 (2 - x)\,dx$$

[redacted]

8. Set up (***Do NOT evaluate***) the integral needed to find the volume of the solid generated by revolving the region bounded by the two curves $x = y^2$ and $y = x$ about the line $x = 1$

Intersection $y^2 = y \to y = 0, 1$

$$V = \int_0^1 \pi\left[(1-y^2)^2 - (1-y)^2\right] dy$$



9. Use the rectangular method (with the left endpoints approximation) to find the Net Signed Area between the graph of $f(x) = 2 - 3x$ and x-axis over $[-1\ ,\ 1]$

$\Delta x = \frac{1+1}{n} = \frac{2}{n}$ $\quad x_k^* = -1 + (k-1)\frac{2}{n} = -1 + \frac{2(k-1)}{n}$

$A = \lim_{n\to\infty} \sum_{k=1}^{n} \left(2 - 3\left(-1 + \frac{2k}{n} - \frac{2}{n}\right)\right)\frac{2}{n} = \lim_{n\to\infty} \frac{2}{n}\sum_{k=1}^{n}\left(2 + 3 - \frac{6k}{n} + \frac{6}{n}\right.$

$= \lim_{n\to\infty} \frac{2}{n}\left[5n - \frac{6}{\cancel{n}}\,\frac{\cancel{n}(n+1)}{2} + 6\right] = \lim_{n\to\infty}\left(10 - \frac{6(n+1)}{n} + \frac{12}{n}\right.$

$= 10 - 6 + 0 = 4$