MATH 002 – T022 (EXAM I) MASTER

Part I: (6-pionts) MULTIPLE CHOICE QUESTIONS (MCQ)

[Bubble the correct answer on the OMR sheet]

1. Which one of the following statements is TRUE for **any nonzero real numbers x and y?**

(a) $\log\left(\frac{x}{y}\right)^2 = 2\log|x| - 2\log|y|$

(b) $\log\sqrt{x^2+y^2} = \log|x| + \log|y|$

(c) $\log 10^{x+y} = |x+y|$

(d) $10^{\log|x+y|} = x+y$

On the Properties of the Logarithmic and Exponential functions.

2. The graph of the function $f(x) = \ln|x|$ is **increasing** on the interval:

(a) $(0,\infty)$

(b) $(-\infty,0)$

(c) $(-\infty,0)\cup(0,\infty)$

(d) $(-1,0)\cup(0,1)$

Problem #64 P.328

3. The length of an arc that subtends a central angle of 99° in a circle of radius 3 feet is equal to:

(a) $\frac{33\pi}{20}$ feet

(b) $\frac{60\pi}{11}$ feet

(c) $\frac{60}{11\pi}$ feet

(d) 297 feet

Similar to Example 5 P. 396

4. If $f(x) = \log_4 x$, then $f^{-1}(\frac{3}{2})$ is equal to:

(a) 8

(b) 6

(c) $\frac{8}{3}$

(d) $2\sqrt[3]{2}$

Similar to Example #1(a) p. 321

5. Given the angles $\alpha = 51°49'$ and $\beta = 38°11'$, then which one of the following statements is TRUE?

(a) α and β are complementary angles

(b) α and β are supplementary angles

(c) $\alpha + \beta$ is an acute angle

(d) $\alpha + \beta$ is an obtuse angle

Similar to Example #1 p.390 & #3 p.399

6. The exact value of $\sin\frac{\pi}{3} - \csc\frac{\pi}{3}\sin\frac{\pi}{6}$ is equal to:

(a) $\frac{\sqrt{3}}{6}$

(b) $\frac{7\sqrt{3}}{6}$

(c) $-\frac{\sqrt{3}}{6}$

(d) $-\frac{2\sqrt{3}}{3}$

Similar to #38 p. 411

Part II: (6-POINTS) TRUE AND FALSE STATEMENTS

[Bubble the correct answer on the OMR sheet. A for TRUE and B for FALSE.]

7. If $\cot\theta < 0$ and $\sec\theta > 0$, then $\sin\theta > 0$.

Similar to #(9 to 14) p. 421

A: TRUE

(B:) FALSE

8. The expression $e^{-1+\ln 2}$ is equal to $\frac{2}{e}$.

(A:) TRUE

B: FALSE

9. If the terminal side of an angle θ contains the point $P(-2,\frac{1}{3})$, then $\sin\theta = \frac{1}{3}$.

Similar to Example #1 p 416 & #6 p. 421

A: TRUE

(B:) FALSE

10. The reference angle of the angle $-30°$ is equal to $330°$.

Similar to Example #3 p. 419 and #6 p. 422

A: TRUE

(B:) FALSE

11. The product $(\log_5 3)(\log_3 5)(\log_5 25)$ is equal to 2.

similar to #72 p. 341

(A:) TRUE

B: FALSE

12. The function $f(x) = x^3 - \tan x$ is an odd function.

Similar to #33 to 40 p. 432

(A:) TRUE

B: FALSE

PART III: WRITTEN QUESTIONS

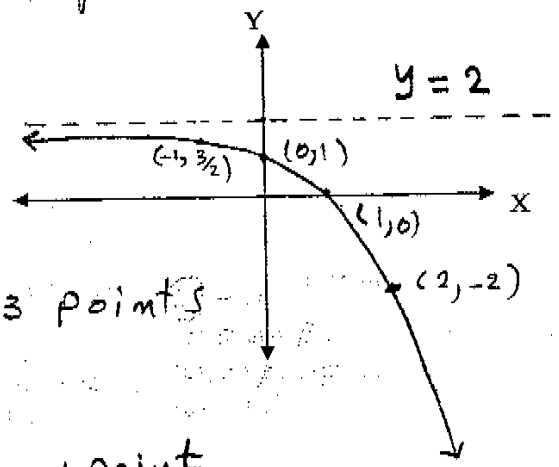
[Provide neat and complete solution to each question. Show necessary steps for full credit.]

1. (6-points) Sketch the graph of the function $f(x)=-2^x+2$, and find its domain, range, x - intercept, y - intercept, and the asymptotes as indicated below:

similar to # 25 & 35 P. 317 and Example 1 p. 312

a) The graph

The graph of $f(x)=-2^x+2$ is the graph of $g(x)=2^x$ reflected across the x-axis and shifted up vertically 2 units



--- 3 points

b) The domain = $(-\infty, \infty)$

The range = $(-\infty, 2)$

} --- 1 point

c) The x - intercept: $f(x)=0 \Rightarrow 2^x=2 \Rightarrow x=1 \Rightarrow$ x-intercept $=(1,0)$

The y - intercept: $x=0 \Rightarrow f(0)=-1+2=1 \Rightarrow$ y-intercept $=(0,1)$

--- 1 point

d) The Asymptotes: $y=2$, No other asymptotes.

--- 1 point

2. (3-points) Find the exact value of $\csc\frac{19\pi}{6}+\tan 144^\circ+\tan 36^\circ$. (Show your steps)

$\frac{19\pi}{6}=3\pi+\frac{\pi}{6} \Rightarrow \frac{19\pi}{6}$ is a Quadrant III angle whose reference angle is $\frac{\pi}{6}$ --- 1 point

and 146° is a Quadrant II angle whose reference angle is $180^\circ-146^\circ=36^\circ$ --- 1 point

$\Rightarrow$ The given expression $=-\csc\frac{\pi}{6}-\tan 36^\circ+\tan 36^\circ=-2$

Similar to Example 4 p. 420 and # 37 & 48 P. 422 --- 1 point

3. (3-points) Write the following expression as a **single natural logarithm**:

$$-2+\frac{1}{3}\ln\left(\frac{x^3}{y^6}\right)+2\ln(xye)$$

The given expression $= \ln e^{-2} + \ln\left(\frac{x^3}{y6}\right)^{1/3} + \ln(xye)^2$ --- 1 point

$= \ln\left[e^{-2}\cdot\frac{x}{y^2}\cdot x^2y^2e^2\right]$ --- 1 point

$= \ln x^3$. --- 1 point

Similar to # 11 to 20 p. 339-340

4. (3-points) Let W be the wrapping function, and t be a real number with $\frac{3\pi}{2} < t < 2\pi$.

If $W(t)=P(\frac{\sqrt{5}}{3},b)$, then find the value of b.

The point $P(\frac{\sqrt{5}}{3}, b)$ lies on the unit circle $x^2+y^2=1$ and in Quadrant IV --- 1 point

$\Rightarrow \frac{5}{9}+b^2=1 \Rightarrow b^2=\frac{4}{9} \Rightarrow b=\pm\frac{2}{3}$ --- 1 point

$\Rightarrow b=-\frac{2}{3}$ only --- 1 point

See the definition of W + # 1 to 12 p. 431

5. (3-points) Express $\tan\theta-\frac{\sec^2\theta}{\tan\theta}$ in terms of $\cot\theta$. (Show your steps).

The expression $= \frac{\tan^2\theta-\sec^2\theta}{\tan\theta}$ --- 1 point

$= \frac{-1}{\tan\theta}$ --- 1 point

$= -\cot\theta$. --- 1 point

similar to #49 to 62 p. 432

MATH 002 – T022 (EXAM I) MASTER

6. (4-points) Solve: $\frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{1}{4}$

$\Rightarrow 4e^x - 4e^{-x} = e^x + e^{-x}$ --- 1 point

$\Rightarrow 3e^x = 5e^{-x}$ --- 1 point

$\Rightarrow e^{2x} = \frac{5}{3}$ --- 1 point

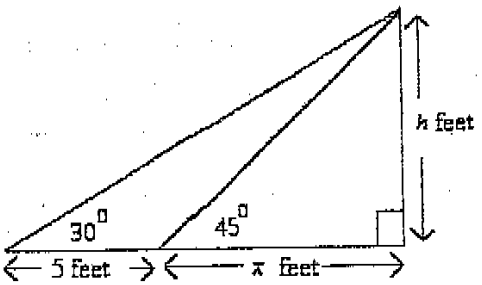
$\Rightarrow 2x = \ln\frac{5}{3} \Rightarrow x = \frac{1}{2}\ln\frac{5}{3}$ --- 1 point

Similar to # 39 to 46 p. 349

7. (3-points) Find the exact value of the height h of the right triangle shown in the figure: (Show your steps)

From the figure $\Rightarrow$

$x = h$ --- 1 point



and $\tan 30^\circ = \frac{h}{5+x}$

$= \frac{h}{5+h} = \frac{1}{\sqrt{3}}$ --- 1 point

$\Rightarrow \sqrt{3}h = 5+h \Rightarrow (\sqrt{3}-1)h = 5 \Rightarrow h = \frac{5}{\sqrt{3}-1}$

--- 1 point

Similar to Example 6 p. 409

MATH 002 -- T022 (EXAM I)

8. (3-points) A wheel is rotating at 100 revolutions per minute. Find the exact angular speed of the wheel in radians per seconds. (show your steps).

$$100 \text{ rev/minute} = \frac{100 \text{ revolutions}}{1 \text{ minute}} \left(\frac{2\pi \text{ radians}}{1 \text{ rev}}\right)\left(\frac{1 \text{ minute}}{60 \text{ seconds}}\right)$$

--- 2 points

similar to #69 p.400 and example 7 p.397

$$= \frac{200\pi}{60} \text{ radians/second}$$

$$= \frac{10\pi}{3} \text{ radians/second} \quad \text{--- 1 point}$$

9. (4-points) Solve the logarithmic equation:

$$\log(x+5) + \log(x-2) = \log(12x)$$

$\Rightarrow \log[(x+5)(x-2)] = \log(12x)$ --- 1 point

$\Rightarrow (x+5)(x-2) = 12x$

$\Rightarrow x^2 + 3x - 10 = 12x$

$\Rightarrow x^2 - 9x - 10 = 0$ --- 1 point

$\Rightarrow (x-10)(x+1) = 0$

$\Rightarrow x = 10$ or $x = -1$ --- 1 point

Check $\Rightarrow$ $x = 10$ is the only solution of the given equation --- 1 point

Similar to #21 to 30 P.349