

**King Fahd University of Petroleum and Minerals**  
**College of Sciences**  
**Prep-Year Math Program**

Code: Master

**Math 002 Exam II**  
**Term 022 (2002-2003)**  
**Sunday, April 27, 2003**  
**Time Allowed: 90 Minutes**

Code: Master

Student's Name: .....  
 ID #: ..... Section #: .....

**This exam consists of two parts:**

**Part I : Multiple Choice**      **Bubble the correct answer on the OMR sheet**  
**Part II : Written**              **Provide neat and complete solutions.**  
    **Show all necessary steps for full credit.**

**Calculators, pagers, or mobiles are NOT allowed during this examination.**

Question	Points	Student's Score	Grader
Part I: Multiple Choice	12		
Part II: Written			
1	8		Mr Al-Absi
2	4		Mr Ahmad
3	3		Dr Al-Attas
4	3		Mr Al-Humaidi
5	3		Mr Maslamani
6	3		Mr Saifullah
7	4		Mr Shehadeh
8	4		Mr Yushau

Total

44

**Part I:** (12-points) MULTIPLE CHOICE QUESTIONS (MCQ)**[Bubble the correct answer on the OMR sheet]**

1. For  $0 \leq x \leq 4\pi$ , the graph of  $y = -3\sec\frac{x}{2}$  lies completely below the  $x$ -axis on the interval:

- a)  $[0, \pi) \cup (3\pi, 4\pi]$
- b)  $(\pi, 3\pi)$
- c)  $[0, \pi) \cup (\frac{3\pi}{2}, 2\pi]$
- d)  $(\frac{\pi}{2}, \frac{3\pi}{2})$

2. The expression  $\sqrt{3}\sin 10^\circ + \cos 10^\circ$  is equal to:

- a)  $2\sin 40^\circ$
- b)  $2\sin 70^\circ$
- c)  $2\sin 100^\circ$
- d)  $4\sin 20^\circ$

3. The expression  $\frac{\sin \theta}{\csc \theta - \cot \theta}$  is identical to:

- a)  $1 + \cos \theta$
- b)  $1 - \cos \theta$
- c)  $1 + \sin \theta$
- d)  $1 - \sin \theta$

4. The domain  $D$  and the range  $R$  of the function  $f(x) = -3\sin^{-1}(2x-1)$  are:

a)  $D = [0,1]$  ;  $R = [-\frac{3\pi}{2}, \frac{3\pi}{2}]$

b)  $D = [-1,1]$  ;  $R = [-\frac{\pi}{2}, \frac{\pi}{2}]$

c)  $D = [-1,1]$  ;  $R = [-\pi, \pi]$

d)  $D = [0,1]$  ;  $R = [-\frac{\pi}{6}, \frac{\pi}{6}]$

5. The expression:  $\sin 13^\circ \sin 73^\circ + \sin 77^\circ \sin 17^\circ$  is equal to:

a)  $\frac{1}{2}$

b)  $\frac{\sqrt{3}}{2}$

c)  $-\frac{1}{2}$

d)  $-\frac{\sqrt{3}}{2}$

6. The number of the vertical asymptotes to the graph of  $y = \tan(\frac{\pi}{2} + 2x)$  over the interval  $[0, \pi]$  is equal to:

a) 3

b) 2

c) 1

d) 4

MATH 002 - T022 (EXAM II)

MASTER

7. Which one of the following statements is FALSE:

a)  $\sin^{-1}\left(\sin\frac{4\pi}{5}\right) = \frac{4\pi}{5}$

b)  $\tan^{-1}\left(\tan\frac{\pi}{5}\right) = \frac{\pi}{5}$

c)  $\cos^{-1}\left(\cos\frac{4\pi}{5}\right) = \frac{4\pi}{5}$

d)  $\tan^{-1}\left(\tan\frac{4\pi}{5}\right) = -\frac{\pi}{5}$

8. The number of solutions of the equation  $\sin^2 x = \frac{1}{2} \sin 2x$  over the interval  $\left[0, \frac{3\pi}{2}\right)$  is equal to:

a) 4

b) 3

c) 2

d) 1

MATH 002 - T022 (EXAM II)

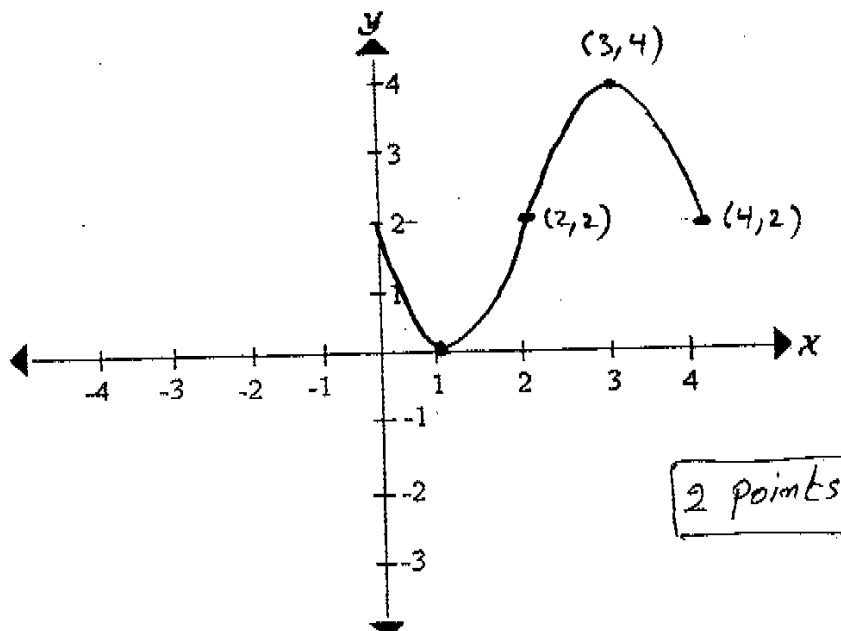
**PART II: WRITTEN QUESTIONS**

Provide neat and complete solution to each question. Show necessary steps for full credit.

1. (8-points) Given the function

$$f(x) = -2 \cos\left(\frac{\pi}{2}x - \frac{\pi}{2}\right) + 2$$

- a) The period of  $f$  is:  $= 2\pi/|b| = 2\pi/(\pi/2) = 4$  --- 1 point
- b) The phase shift of the graph of  $f$  is:  $= -c/b = -(-\pi/2)/(\pi/2) = 1$  --- 1 point
- c) The vertical translation of the graph of  $f$  is:  $= 2$  units up --- 1 point
- d) The amplitude of the graph of  $f$  is:  $= |a| = |-2| = 2$  --- 1 point
- e) The domain of  $f$  is:  $= (-\infty, \infty)$  --- 1 point
- f) The range of  $f$  is:  $= [0, 4]$  --- 1 point
- g) Use all the above to sketch the graph of  $f$  over the interval  $[0, 4]$ .

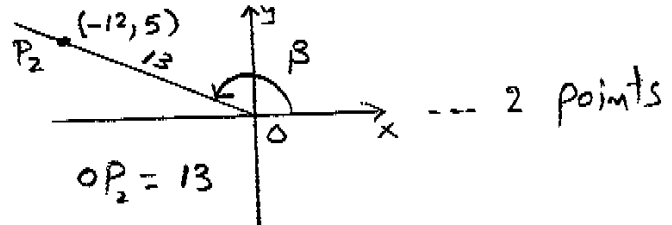
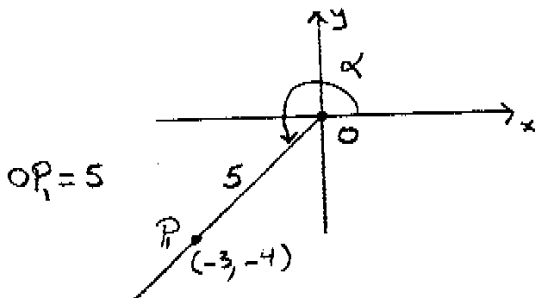


MATH 002 - T022 (EXAM II)

2. (4-points) Given  $\sin \alpha = -\frac{4}{5}$ ,  $\alpha$  in Quadrant III, and  $\cos \beta = -\frac{12}{13}$ ,  $\beta$  in Quadrant II.

Find the exact value of  $\cot(\alpha + \beta)$

$\alpha$  and  $\beta$  are as given in the following figures:



$$\begin{aligned} \Rightarrow \cot(\alpha + \beta) &= \frac{1}{\tan(\alpha + \beta)} = \frac{1 - \tan \alpha \tan \beta}{\tan \alpha + \tan \beta} \quad \dots 1 \text{ point} \\ &= \frac{1 - \left(\frac{4}{3}\right)\left(-\frac{5}{12}\right)}{\frac{4}{3} + \left(-\frac{5}{12}\right)} \\ &= \frac{36 + 20}{48 - 15} = \frac{56}{33} \quad \dots 1 \text{ point} \end{aligned}$$

3. (3-points) Verify the identity  $\frac{\cos 2x + \cos x}{\sin 2x - \sin x} = \csc x + \cot x$

$$\text{LHS} = \frac{\cos 2x + \cos x}{\sin 2x - \sin x} = \frac{2\cos^2 x - 1 + \cos x}{2\sin x \cos x - \sin x} \quad \dots 1 \text{ point}$$

$$= \frac{(2\cos x - 1)(1 + \cos x)}{\sin x(2\cos x - 1)} = \frac{1 + \cos x}{\sin x} \quad \dots 1 \text{ point}$$

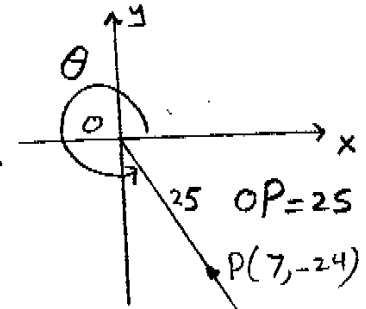
$$= \frac{1}{\sin x} + \frac{\cos x}{\sin x} = \csc x + \cot x = \text{RHS} \quad \dots 1 \text{ point}$$

MATH 002 - T022 (EXAM II)

4. (3-points) If  $\csc \theta = -\frac{25}{24}$ ,  $\frac{7\pi}{2} < \theta < 4\pi$ , find the exact value of  $\sin \frac{\theta}{2}$

$\frac{7\pi}{2} < \theta < 4\pi \Rightarrow \frac{7\pi}{4} < \frac{\theta}{2} < 2\pi \Rightarrow \theta$  and  $\frac{\theta}{2}$  are both in Quadrant IV and  $\theta$  as in the figure } ... 1.5 points

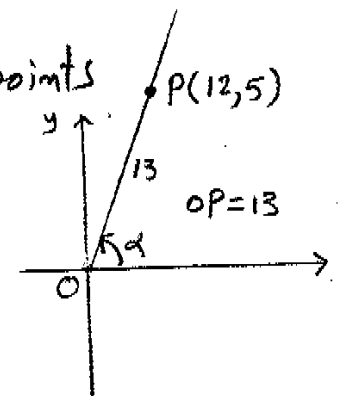
$$\begin{aligned} \Rightarrow \sin \frac{\theta}{2} &= -\sqrt{\frac{1 - \cos \theta}{2}} \\ &= -\sqrt{\frac{1 - \frac{7}{25}}{2}} = -\sqrt{\frac{18}{50}} \\ &= -\sqrt{\frac{9}{25}} = -\frac{3}{5} \end{aligned}$$



5. (3-points) Solve the inverse trigonometric equation:  $\cos^{-1} x + \tan^{-1} \frac{5}{12} - \frac{\pi}{2} = 0$

$$\begin{aligned} \Rightarrow \cos^{-1} x &= \frac{\pi}{2} - \tan^{-1} \frac{5}{12} \\ \Rightarrow \cos(\cos^{-1} x) &= \cos\left(\frac{\pi}{2} - \tan^{-1} \frac{5}{12}\right) \\ \Rightarrow x &= \sin\left(\tan^{-1} \frac{5}{12}\right) \end{aligned}$$

Let  $\alpha = \tan^{-1} \frac{5}{12}$  } ... 1.5 points  
 $\Rightarrow \alpha$  as in the figure } ... 1.5 points  
 $\Rightarrow x = \sin \alpha = \frac{5}{13}$

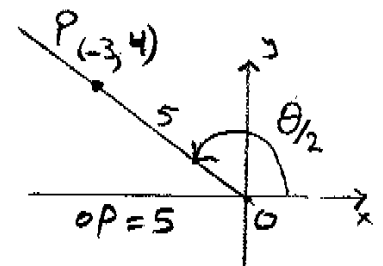


6. (3-points) If  $\cos \frac{\theta}{2} = -\frac{3}{5}$ ,  $\pi < \theta < \frac{3\pi}{2}$ , find the exact value of  $\tan \theta$

$\pi < \theta < \frac{3\pi}{2} \Rightarrow \frac{\pi}{2} < \frac{\theta}{2} < \frac{3\pi}{4} \Rightarrow \frac{\theta}{2}$  is in Quadrant II and as given in the figure } ... 1.5 points

$$\begin{aligned} \Rightarrow \tan \theta &= \frac{2 \tan \frac{\theta}{2}}{1 - \tan^2 \frac{\theta}{2}} = \frac{2(-\frac{4}{3})}{1 - (-\frac{4}{3})^2} \\ &= \frac{-24}{9 - 16} = \frac{24}{7} \end{aligned}$$

**OR**  $\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}}{2 \cos^2 \frac{\theta}{2} - 1} \dots \text{etc.}$



## MATH 002 - T022 (EXAM II)

7. (4-points) Find the exact solutions of the equation  $\tan \frac{x}{2} = \sin x$ ,  $0 \leq x < 2\pi$

$$\tan \frac{x}{2} = \sin x \Rightarrow \frac{1 - \cos x}{\sin x} = \sin x \quad \dots 1 \text{ point}$$

$$\Rightarrow \left. \begin{aligned} 1 - \cos x &= \sin^2 x \\ 1 - \cos x &= 1 - \cos^2 x \\ \cos x &= \cos^2 x \end{aligned} \right\} \dots 1 \text{ point}$$

$$\Rightarrow \left. \begin{aligned} \cos x (\cos x - 1) &= 0 \\ \cos x = 0 &\text{ or } \cos x = 1 \end{aligned} \right\} \dots 1 \text{ point}$$

$$\Rightarrow \left. \begin{aligned} x = \frac{\pi}{2}, \frac{3\pi}{2} &\text{ or } x = 0 \end{aligned} \right\} \dots 1 \text{ point}$$

$$\Rightarrow \text{The solutions are: } 0, \frac{\pi}{2}, \frac{3\pi}{2}$$

Important Note: Some students may use  $\frac{\sin x}{1 + \cos x} = \sin x$

$\Rightarrow$  Possible solutions are  $0, \pi, \frac{\pi}{2}, \frac{3\pi}{2}$

$\Rightarrow \pi$  must be rejected since  $\tan \frac{\pi}{2}$  is undefined.

8. (4-points) Find the exact value of the expression:  $\left( \cos \frac{4\pi}{9} - \cos \frac{\pi}{9} \right)^2 + \left( \sin \frac{4\pi}{9} - \sin \frac{\pi}{9} \right)^2$

$$\text{The expression} = \left. \begin{aligned} &\cos^2 \frac{4\pi}{9} - 2 \cos \frac{4\pi}{9} \cos \frac{\pi}{9} + \cos^2 \frac{\pi}{9} \\ &+ \sin^2 \frac{4\pi}{9} - 2 \sin \frac{4\pi}{9} \sin \frac{\pi}{9} + \sin^2 \frac{\pi}{9} \end{aligned} \right\} \dots 1 \text{ point}$$

$$= \left. \begin{aligned} &(\cos^2 \frac{4\pi}{9} + \sin^2 \frac{4\pi}{9}) + (\cos^2 \frac{\pi}{9} + \sin^2 \frac{\pi}{9}) \\ &- 2 \left( \cos \frac{4\pi}{9} \cos \frac{\pi}{9} + \sin \frac{4\pi}{9} \sin \frac{\pi}{9} \right) \end{aligned} \right\} \dots 1 \text{ point}$$

$$\begin{aligned} &= 1 + 1 - 2 \cos \left( \frac{4\pi}{9} - \frac{\pi}{9} \right) \\ &= 2 - 2 \cos \frac{\pi}{3} \\ &= 2 - 2 \left( \frac{1}{2} \right) = 1 \end{aligned} \left. \right\} \dots 2 \text{ points}$$