9.3 Nonlinear Systems of Equations

- A nonlinear system of equations is one or more equations of the system are not linear.

Example #1 Solve

$$x^2 + 3y^2 = 7 \qquad (1)$$

$$x + 4y = 6 \qquad (2)$$

Solution

Solve Eq.(2) for x

$$x = 6 - 4y$$

Substitute $6 - 4y$ for x in Eq.(1)

$$(6 - 4y)^2 + 3y^2 = 7$$

Solve for y

$$36 - 48y + 16y^2 + 3y^2 = 7$$

$$19y^2 - 48y + 29 = 0$$

$$(19y - 29)(y - 1) = 0$$

$$y = \frac{29}{19} \qquad y = 1$$

Substitute y in Eq.(2)

$$x + 4\left(\frac{29}{19}\right) = 6 \qquad x + 4(1) = 6$$

$$x = 6 - \frac{116}{19} \qquad x = 6 - 4$$

$$x = \frac{144}{19} - \frac{116}{19} \qquad\qquad x = 2$$

$$x = -\frac{2}{19}$$

The solutions are $\left(-\frac{2}{19}, \frac{29}{19}\right)$ *and* $(2,1)$

Example #2 solve

$$x^2 - 2y^2 = 8 \qquad (1)$$

$$x^2 + 3y^2 = 28 \qquad (2)$$

Solution

To eliminate x^2, multiply Eq.(1) by –1 and add to Eq.(2)

$$-x^2 + 2y^2 = -8$$

$$x^2 + 3y^2 = 28$$

$$0 \;\; + 5y^2 = 20$$

$$y^2 = 4$$

$$y = \pm 2$$

Substitute for y in Eq.(1)

$$x^2 - 2(2)^2 = 8 \qquad\qquad x^2 - 2(-2)^2 = 8$$

$$x^2 = 8 + 8 \qquad\qquad x^2 = 8 + 8$$

$$x^2 = 16 \qquad\qquad x^2 = 16$$

$$x^2 = \pm 4 \qquad\qquad x^2 = \pm 4$$

The solutions are $(4,2)$, $(4,-2)$, $(-4,2)$ *and* $(-4,-2)$.

Example #3 solve

$$2x^2 + 3y^2 = 5 \qquad (1)$$

$$x^2 - 3y^2 = 4 \qquad (2)$$

Solution

To eliminate y^2, add Eq.(1) and Eq.(2)

$$2x^2 + 3y^2 = 5$$

$$\underline{x^2 - 3y^2 = 4}$$

$$3x^2 + 0 \quad = 9$$

$$x^2 = 3$$

$$x = \pm\sqrt{3}$$

Substitute for y in Eq.(2)

$$\left(\sqrt{3}\right)^2 - 3y^2 = 4$$

$$-3y^2 = 4 - 3$$

$$y^2 = -\frac{1}{3}$$

No real Solution

The graph of the equations does not intersect.