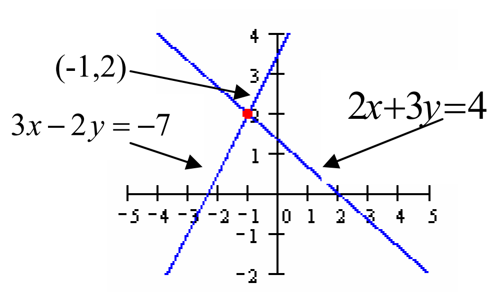
CHAPTER 9
SYSTEMS OF EQUATIONS

9.1 Systems of Linear Equations in Two Variables

- A system of equations is two or more equations considered together.
- The following system of equations is a linear system of equations in two variables.

$$2x+3y=4$$
$$3x-2y=-7$$

- A solution of a system of equations in two variables is an ordered pair that is a solution of both equations, and intersection of the graph of the two lines.



- If the graphs of the two lines are parallel, the system is called inconsistent system and has no solution.
- If the graphs of the two lines are intersect at single point (independent system) or are the same line (dependent system), the system called consistent system.

Substitution Method for Solving a System of Linear Equations

Example #1 Solve

$$\begin{aligned} 2x+3y&=4 \qquad (1)\\ 3x-2y&=-7 \qquad (2) \end{aligned}$$

Solution

Solve Eq. (1) for y

$$\rightarrow y=\frac{4}{3}-\frac{2}{3}x \qquad (3)$$

Substitute $\frac{4}{3}-\frac{2}{3}x$ for y in Eq. (2)

$$3x-2\left(\frac{4}{3}-\frac{2}{3}x\right)=-7$$

Solve for x

$$3x-\frac{8}{3}+\frac{4}{3}x=-7$$

$$3x+\frac{4}{3}x=-7+\frac{8}{3}$$

$$\frac{13}{3}x=\frac{-13}{3}$$

$$x=-1$$

Substitute –1 for x in Eq. (3)

$$\rightarrow y=\frac{4}{3}-\frac{2}{3}(-1)=\frac{4}{3}+\frac{2}{3}=\frac{6}{3}=2$$

The solution is $(-1,2)$.

Elimination Method for Solving a System of Linear Equations

Example #2 Solve

$$2x + 3y = 4 \qquad (1)$$
$$3x - 2y = -7 \qquad (2)$$

Solution

To eliminate the variable x multiply each side of Eq. (1) by 3, and each side of Eq. (2) by –2, and add the equations

$$\begin{array}{r} 6x + 9y = 12 \\ \underline{-6x + 4y = 14} \\ 0 + 13y = 26 \\ y = 2. \end{array}$$

Substitute 2 for y in Eq. (1) and solve for x

$$\begin{array}{r} 2x + 3(2) = 4 \\ 2x = -2 \\ x = -1 \end{array}$$

The solution is $(-1, 2)$.

Example #3 Solve

$$2x - y = 3 \qquad (1)$$
$$-4x + 2y = 4 \qquad (2)$$

Solution

To eliminate y multiply each side of Eq. (1) by 2, and add the result to Eq. (2).

$$\begin{aligned} 4x - 2y &= 6 \\ -4x + 2y &= 4 \\ \hline 0 + 0 &= 10 \end{aligned}$$

$$0 = 10, \qquad \text{* A false equation}$$

Then no solution (inconsistent system)

Example #4 Solve

$$4x + 5y = 2 \qquad (1)$$

$$12x + 15y = 6 \qquad (2)$$

Solution

To eliminate x, multiply each side of Eq. (1) by –3, and add the result to Eq. (2)

$$\begin{aligned} -12x - 15y &= -6 \\ 12x + 15y &= 6 \\ \hline 0 + 0 &= 0 \end{aligned}$$

$$0 = 0 \qquad \text{* A true equation}$$

Then the system has infinite number of solutions (dependent system)

Substitute any real number c for x in Eq. (1) and solve for y

$$4c + 5y = 2$$

$$5y = 2 - 4c$$

$$y = \frac{2}{5} - \frac{4}{5}c$$

The solution is $\left(c, \frac{2}{5} - \frac{4}{5}c \right)$.

Example #4 Find the values of a and b where the system

$$3x - 2ay = 4$$

$$2x + 3y = b.$$

A) is inconsistent

B) is independent.

C) Evaluate $3b - 4a$ where the system is dependent.

Solution:

To eliminate x by multiplying eq.(1) by -2 and eq.(2) by 3

$$-6x + 4ay = -8$$

$$\underline{6x + 9y = 3b.}$$

$$0 + 4ay + 9y = 3b - 8$$

$$(4a + 9)y = 3b - 8$$

A)

$$4a + 9 = 0 \quad and \quad 3b - 8 \neq 0$$

$$a = -\frac{9}{4} \quad and \quad b \neq \frac{8}{3}$$

B) $a \neq -\frac{9}{4}$, b can be any real number

C) $3b - 4a = 3\frac{8}{3} - 4(-\frac{9}{4}) = 17$