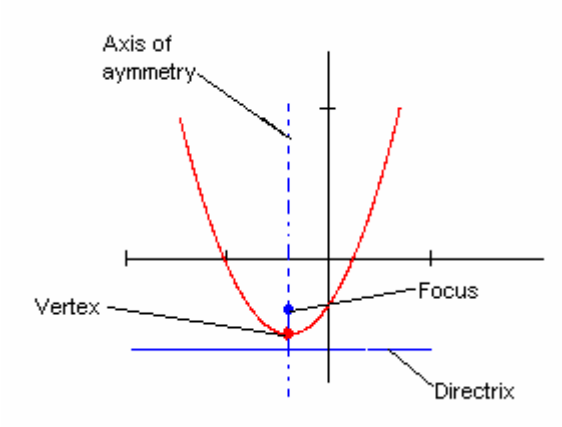
CHAPTER 8
Topic in Analytic Geometry

8.1 Parabolas



Definition of Parabola

A **parabola** is the set of points in the plane that are equidistant from a fixed line (the **directrix**) and fixed point (the **focus**) not in the directrix.

Standard Forms of the Equation of a Parabola with Vertex at (h,k)

- Vertical Axis of Symmetry

$$(x-h)^2 = 4p(y-k)$$

The focus is $(h, k+p)$, and the equation of the directrix is $y = k - p$.

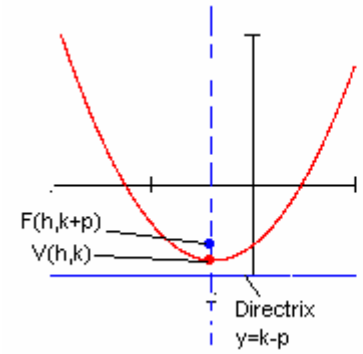
* If p>0 the parabola opens up. If p<0 the parabola opens down.

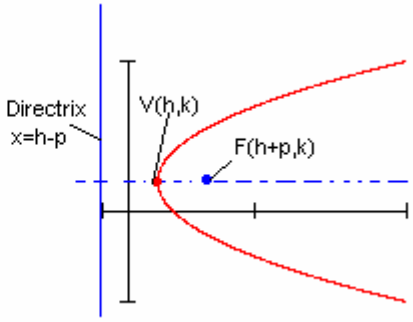
- Horizontal Axis of Symmetry

$$(y-k)^2 = 4p(x-h)$$

The focus is $(h+p,k)$, and the equation of the directrix Is $x = h - p.$

* If p>0 the parabola opens to the right. If p<0 parabola opens to the left.





Example #1 Find the vertex, focus, directrix, and axis of symmetry of each parabola. Sketch the graph.

a) $(y+4)^2 = -4x+8$

Solution

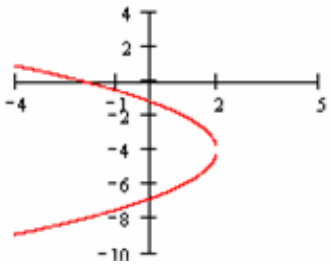
horizontal parabola

$(y+4)^2 = -4(x-2), \quad vertex = (2,-4)$

$4p = -4 \rightarrow p = -1, \quad open\ to\ the\ left$

$focus\,(h+p,k) = (2-1,-4) = (1,-4)$

$directrix\ x = h - p = 2+1 = 3$



b) $4x^2-12x+12y+7=0$

Solution

Vertical parabola

First rewrite the equation in standard form,

by complete the square on x.

$4x^2-12x+12y+7=0, \quad 4x^2-12x=-12y-7$

$4(x^2-3x)=-12y-7$

$4\left[x^2-3x+\left(\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2\right]=-12y-7,\ 4\left[\left(x-\frac{3}{2}\right)^2-\frac{9}{4}\right]=-12y-7$

$4\left(x-\frac{3}{2}\right)^2-9=-12y-7$

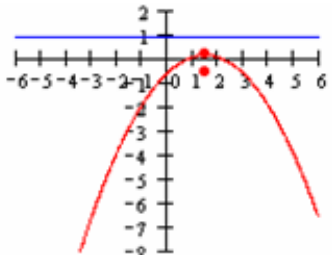
$4\left(x^2-\frac{3}{2}\right)^2=-12y+2, \quad 4\left(x-\frac{3}{2}\right)^2=-12\left(y-\frac{1}{6}\right)$

$\left(x-\frac{3}{2}\right)^2=-3\left(y-\frac{1}{6}\right), \quad vertex\ \left(\frac{3}{2},\frac{1}{6}\right)$

$4p=-3\rightarrow p=-\frac{3}{4}$

$focus\ (h,k+p)=\left(\frac{3}{2},\frac{1}{6}-\frac{3}{4}\right)=\left(\frac{3}{2},-\frac{7}{12}\right)$

$directrix\ y=k-p=\frac{1}{6}-(-\frac{3}{4})=\frac{11}{12}$



c) $2x - y^2 - 6y + 1 = 0$

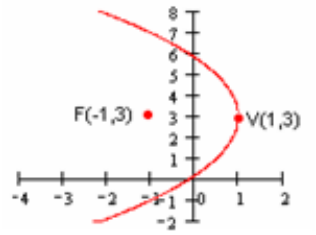
H.W.

Do exr. 3, 11, 22, and 24, page 592.

Example #2 Find the equation in standard form of the parabola with vertex at (1,3) and focus at (-1,3).

Solution

$focus\,(-1,3), vertex\,(1,3)$



horizontal parabola open to the left

$$(y-k)^2 = 4p(x-h)$$

$p = the$ distance *between the vertex and the focus*

$= 1-(-1) = 2$

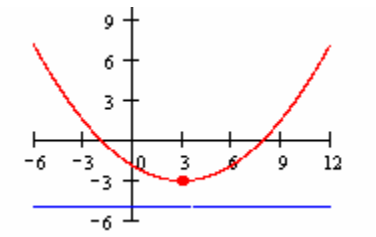
Then the equation of the parabola is

$$(y-3)^2 = 4(-2)(x-1) \qquad * p \text{ is negative because the}$$

or $$(y-3)^2 = -8(x+1) \qquad \text{parabola open to the left.}$$

Example # 3 Find the equation in standard form of the parabola with vertex at (3,-3) and directrix y = -5.

Solution



Vertical parabola open up

$$(x-h)^2 = 4p(y-k)$$

$p = the$ distance $between\ the\ vertex\ and\ the\ directrix$

$= -3-(-5) = 2$

Then the equation of the parabola is

$$(x-3)^2 = 4(2)(y+3) \qquad * p \text{ is positive because}$$

$$or\ (x-3)^2 = 8(y+3) \qquad \text{parabola open up.}$$

- Do exr. 29,32 and 34 page 592.