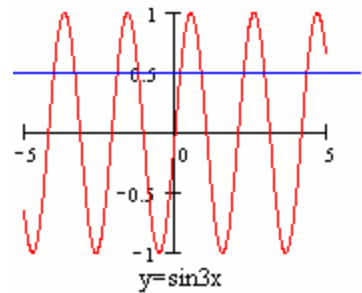


## 6.6 Trigonometric Equations

**Example #1** Solve:  $\sin 3x = \frac{1}{2}$

**Solution**



$\sin 3x = \frac{1}{2}$ , implies

*Quadrant I*

$$3x = \frac{\pi}{6} + 2k\pi, \quad k \text{ an integer}$$

$$x = \frac{\pi}{18} + \frac{2k\pi}{3}$$

*Quadrant II*

$$3x = \frac{5\pi}{6} + 2k\pi, \quad k \text{ an integer}$$

$$x = \frac{5\pi}{18} + \frac{2k\pi}{3}$$

The solutions are  $\frac{\pi}{18} + \frac{2k\pi}{3}, \frac{5\pi}{18} + \frac{2k\pi}{3}$ .

**Example #2** Solve each equation for exact solution in the interval  $0 \leq x < 2\pi$ .

a)  $2 \sin x \cos x = \sin x$

$$2 \sin x \cos x - \sin x = 0$$

$$\sin x(2 \cos x - 1) = 0$$

$$\sin x = 0 \quad 2 \cos x - 1 = 0$$

$$x = 0, \pi \quad \cos x = \frac{1}{2}$$

$$x = \frac{\pi}{3}, 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$$

*\* Quadrant I & IV*

The solutions are  $0, \pi, \frac{\pi}{3}, \frac{5\pi}{3}$ .

b)  $\tan^2 x - \tan x = 2$

**Solution**

$$\tan^2 x - \tan x - 2 = 0$$

$$(\tan x - 2)(\tan x + 1) = 0$$

$$\tan x - 2 = 0 \quad \tan x + 1 = 0$$

$$\tan x = 2 \quad \tan x = -1$$

$$x = \tan^{-1} 2. \quad x = \pi - \frac{\pi}{4} = \frac{3\pi}{4}, 2\pi - \frac{\pi}{4} = \frac{7\pi}{4} \text{ * Quadrant II \& IV}$$

The solutions are  $\frac{3\pi}{4}, \frac{7\pi}{4}, \tan^{-1} 2$

c)  $\cos 2x = 1 - 3 \sin x$

**Solution**

$$\cos 2x = 1 - 3 \sin x$$

$$1 - 2 \sin^2 x = 1 - 3 \sin x$$

$$0 = 2 \sin^2 x - 3 \sin x$$

$$0 = \sin x(2 \sin x - 3)$$

$$\sin x = 0 \quad 2 \sin x - 3 = 0$$

$$x = 0, \pi \quad \sin x = \frac{3}{2}$$

No solution.

The solutions are  $0, \pi$ .

d)  $2 \sin 3x - \csc 3x = -1$

**Solution**

$$2 \sin 2x - \frac{1}{\sin 3x} = -1$$

$$2 \sin^2 2x - 1 = -\sin 2x \quad \text{* Multiply each side by } \sin 2x$$

Continue H.W.

Do exr. 8, 19, 57, 75 Page 524.

Example # 3 Find the exact solution

$$a) \sin\left(2x + \frac{\pi}{9}\right) = -\frac{\sqrt{3}}{2}$$

**Solution**

$$\sin\left(2x + \frac{\pi}{9}\right) = -\frac{\sqrt{3}}{2}$$

*Quadrant III*

*Quadrant IV*

$$2x + \frac{\pi}{9} = \left(\pi + \frac{\pi}{3}\right) + 2k\pi$$

$$2x + \frac{\pi}{9} = \left(2\pi - \frac{\pi}{3}\right) + 2k\pi$$

$$2x = -\frac{\pi}{9} + \frac{4\pi}{3} + 2k\pi$$

$$2x = -\frac{\pi}{9} + \frac{5\pi}{3} + 2k\pi$$

$$2x = \frac{11\pi}{9} + 2k\pi$$

$$2x = \frac{14\pi}{9} + 2k\pi$$

$$x = \frac{11\pi}{18} + k\pi$$

$$x = \frac{7\pi}{9} + k\pi$$

$$b) \cos 2x \cos x + \sin 2x \sin x = -1$$

**Solution**

$$\cos 2x \cos x + \sin 2x \sin x = -1$$

$$\cos(2x - x) = -1$$

$$\cos x = -1$$

$$x = \pi + 2k\pi$$

$$x = (1 + 2k)\pi$$

c) See example#2 page 517 and solve

$$\cos 2x + \sin 2x = 1, \quad 0 \leq x < 2\pi$$

**H.W.**

Do exr. 65, 69, 72, 83, page 525.