6.4 Function of the Form $f(x) = a\sin x + b\cos x$

The function given by $f(x) = a\sin x + b\cos x$ can be written in the form $f(x) = k\sin(x+\alpha)$. Where

$k = \sqrt{a^2 + b^2}$, $\cos\alpha = \dfrac{a}{\sqrt{a^2+b^2}}$, and $\sin\alpha = \dfrac{b}{\sqrt{a^2+b^2}}$

$a\sin x + b\cos x = k\sin(x+\alpha)$.

Example #1 Find the amplitude, phase shift, and period, and then graph

a) $f(x) = -\sin x + \sqrt{3}\cos x$ b) $g(x) = \sin\dfrac{x}{3} - \cos\dfrac{x}{3}$

Solution

a) $f(x) = -\sin x + \sqrt{3}\cos x = k\sin(x+\alpha)$

$k = \sqrt{(-1)^2 + (\sqrt{3})^2} = \sqrt{4} = 2$

$\cos\alpha = \dfrac{-1}{2}$, $\sin\alpha = \dfrac{\sqrt{3}}{2}$, *and* α is in Quadrant II

$\alpha' = \dfrac{\pi}{3} \to \alpha = \pi - \dfrac{\pi}{3} = \dfrac{2\pi}{3}$.

Then $f(x) = 2\sin(x + \dfrac{2\pi}{3})$

Amplitude 2, period $P = 2\pi$, phase Shift $-\dfrac{c}{b} = -\dfrac{2\pi}{3}$ to the left.

First we graph $y = 2\sin x$, then shift $\dfrac{2\pi}{3}$ to the left.