CHAPTER 6
Trigonometric Identities and Equations

6.1 Verification of Trigonometric Identities

Fundamental Trigonometric Identities

Reciprocal Identities	$\csc x = \dfrac{1}{\sin x}$	$\sec x = \dfrac{1}{\cos x}$	$\cot x = \dfrac{1}{\tan x}$
Ratio Identities	$\tan x = \dfrac{\sin x}{\cos x}$	$\cot x = \dfrac{\cos x}{\sin x}$	
Pythagorean Identities	$\sin^2 x + \cos^2 x = 1$	$1 + \tan^2 x = \sec^2 x$	$1 + \cot^2 x = \csc^2 x$

Notes To verify an identity

- If one side of the identity is more complex than other, then try to simplify it until it becomes equal to the other side.
- If two sides are complex, then try to simplify both until they become equal.

Example #1 Verify each identities

a) $\cos^2 x - \sin^2 x = 1 - 2\sin^2 x$

Solution

$$L.H.S = (1 - \sin^2 x) - \sin^2 x$$
$$= 1 - 2\sin^2 x = R.H.S$$

b) $\left(\tan x+1\right)^2=\sec^2 x+2\tan x$

Solution

$$\begin{aligned} L.H.S &= \tan^2 x+2\tan x+1 \\ &= (1+\tan^2 x)+2\tan x \\ &= \sec^2 x+2\tan x = R.H.S. \end{aligned}$$

c) $\dfrac{1-\tan^4 x}{\sec^2 x}=1-\tan^2 x$

Solution

$$\begin{aligned} L.H.S &= \frac{(1-\tan^2 x)(1+\tan^2 x)}{\sec^2 x} \\ &= \frac{\sec^2 x(1-\tan^2 x)}{\sec^2 x} \\ &= 1-\tan^2 x = R.H.S. \end{aligned}$$

d) $\sec x=\dfrac{\cot x+\tan x}{\csc x}$

Solution

$$\begin{aligned} R.H.S &= \frac{\cot x+\tan x}{\csc x} = \frac{\dfrac{\cos x}{\sin x}+\dfrac{\sin x}{\cos x}}{\dfrac{1}{\sin x}} \\ &= \frac{\dfrac{\cos^2 x+\sin^2 x}{\sin x\cos x}}{\dfrac{1}{\sin x}} \end{aligned}$$

$$= \frac{1}{\sin x \cos x} \cdot \frac{\sin x}{1}$$

$$= \frac{1}{\cos x} = \sec x = L.H.S.$$

e) $\frac{\sin x}{1-\cos x} = \csc x + \cot x$

Solution

$$L.H.S. = \frac{\sin x}{1-\cos x} \cdot \frac{1+\cos x}{1+\cos x}$$

$$= \frac{\sin x(1+\cos x)}{1-\cos^2 x}$$

$$= \frac{\sin x(1+\cos x)}{\sin^2 x}$$

$$= \frac{(1+\cos x)}{\sin x}$$

$$= \frac{1}{\sin x} + \frac{\cos x}{\sin x}$$

$$= \csc x + \cot x = R.H.S.$$

f) $\frac{1-\sin x}{1+\sin x} - \frac{1+\sin x}{1-\sin x} = -4\sec x \tan x$

Solution

$$L.H.S. = \frac{1-\sin x}{1+\sin x} - \frac{1+\sin x}{1-\sin x}$$

$$= \frac{(1-\sin x)^2 - (1+\sin x)^2}{(1+\sin x)(1-\sin x)}$$

$$= \frac{1 - 2\sin x + \sin^2 x - (1 + 2\sin x + \sin^2 x)}{1 - \sin^2 x}$$

$$= \frac{-4\sin x}{\cos^2 x}$$

$$= -\frac{4\sin x}{\cos x \cos x}$$

$$= -4\frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}$$

$$= -4\tan x \cdot \sec x = R.H.S.$$

g) $\dfrac{\sin x(\tan x + 1) - 2\tan x \cos x}{\sin x - \cos x} = \tan x$

Solution

$$L.H.S. = \frac{\sin x(\tan x + 1) - 2\tan x \cos x}{\sin x - \cos x}$$

$$= \frac{\sin x\left(\dfrac{\sin x}{\cos x} + 1\right) - 2\dfrac{\sin x}{\cos x} \cdot \cos x}{\sin x - \cos x}$$

$$= \frac{\dfrac{\sin^2 x}{\cos x} + \sin x - 2\sin x}{\sin x - \cos x}$$

$$=\frac{\dfrac{\sin^2 x}{\cos x}-\sin x}{\sin x-\cos x}$$

$$=\frac{\dfrac{\sin^2 x-\sin x\cos x}{\cos x}}{\sin x-\cos x}$$

$$=\frac{\sin x\left(\dfrac{\sin x-\cos x}{\cos x}\right)}{\sin x-\cos x}$$

$$=\frac{\dfrac{\sin x}{\cos x}\left(\sin x-\cos x\right)}{\sin x-\cos x}$$

$$=\frac{\sin x}{\cos x}=\tan x=R.H.S.$$