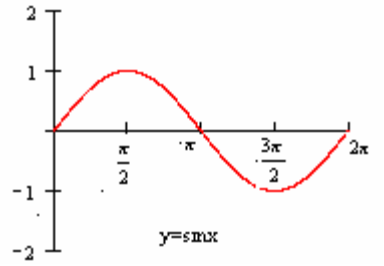
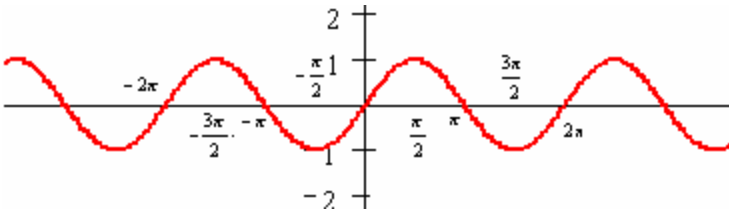
5.5 Graphs of the Sine and Cosine Functions

Example #1 Graph $y = \sin x$

Table 6.7

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$y = \sin x$	0	1	0	-1	0





Definition of a Periodic Function

A function is said to be **periodic** if there exists a positive number p such that $f(x+p) = f(x)$ for all x in the domain f. If P is the smallest value of p, then P is called the **period** of f.

Definition of Amplitude

Suppose f is a periodic function that has a maximum value of M and a minimum value of m. The **amplitude (Amp)** of f, is defined

$$Amp = \frac{M - m}{2}$$

Note: Period of $y = \sin x$ is 2π, and amplitude is 1.

Grapg of $y = a\sin bx,\ b > 0$

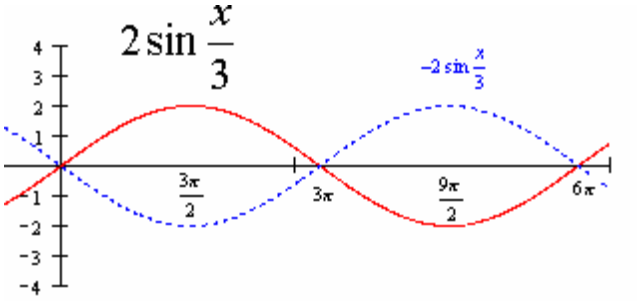
- The amplitude is |a|.
- The period is $\frac{2\pi}{b}$.
- The graph crosses the x-axis at the beginning (0), middle (π / b), and end ($2\pi / b$) of atypical cycle.
- The maximum value occurs at first quarter ($\frac{\pi}{2b}$), and the minimum at third quarter ($\frac{3\pi}{2b}$).
- If a<0, the graph is reflected across the x-axis.

Example# 2 Graph a) $y1 = 2\sin\frac{x}{3}$ b) $y = -2\sin\frac{x}{3}$

Solution

$\text{Amp} = |2| = 2,\ \text{Period}\quad P = \frac{2\pi}{1/3} = 6\pi\ \ \text{start at } 0, \text{middle at } 3\pi, \text{end at } 6\pi$

for y1 maximum at $\frac{3\pi}{2}$ and minimum at $\frac{9\pi}{2}$.

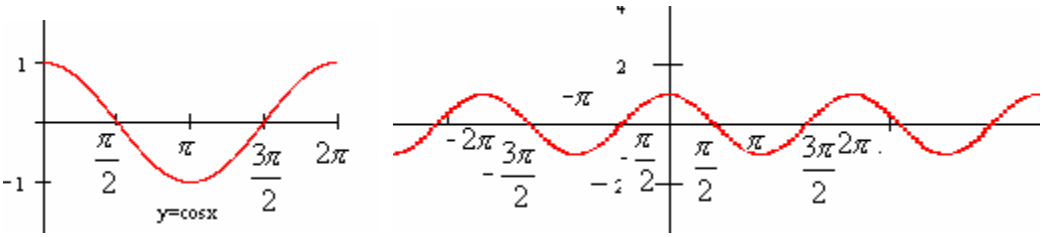


Do exr. 26, 31, and 45 page 441.

Example# 3 Graph $y = \cos x$

Table 6.8

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$y = \cos x$	1	0	-1	0	1



Note: Period of $y = \cos x$ is 2π, and amplitude is 1.

Grapg of $y = a\cos bx,\ b > 0$

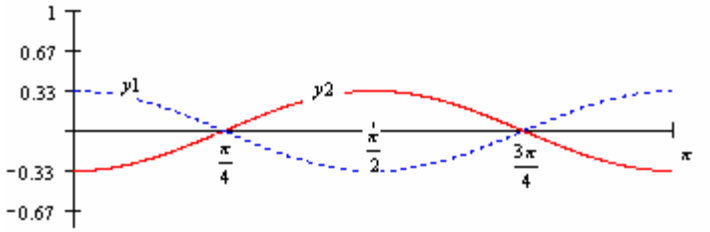
- The amplitude is |a|.
- The period is $\frac{2\pi}{b}$.
- The graph crosses the x-axis at first quarter ($\frac{\pi}{2b}$), and third quarter ($\frac{3\pi}{2b}$).
- The maximum value occurs at start (0) and end ($\frac{2\pi}{b}$), the minimum at middle (π / b).
- If a<0, the graph is reflected across the x-axis.

Example# 4 Graph a) $y1=\frac{1}{3}\cos 2x$ b) $y2=-\frac{1}{3}\cos 2x$

Solution

Amp $=|1/3|=1/3$, Period $P=\frac{2\pi}{2}=\pi$ start at 0, middle at π/2, end at π

for y1 maximum at 0 and π minimum at $\pi/2$. $x-$intercept $\pi/4$ and $3\pi/4$

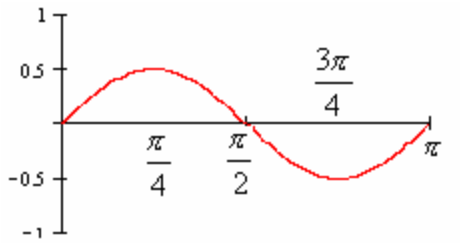


Do exr.28, 42 and 36, page 441.

Example# 5 Graph $y=\left|\frac{1}{2}\sin 2x\right|$, at least one full period

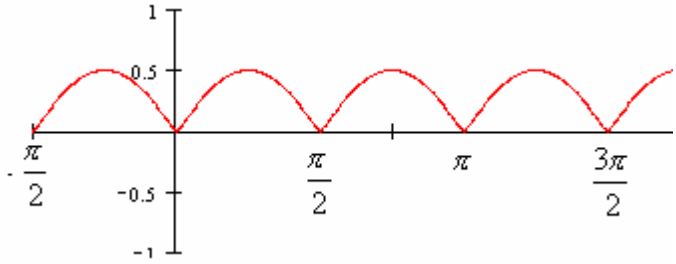
Solution

First we graph y=$\frac{1}{2}\sin 2x$, period $P=\frac{2\pi}{2}=\pi$, amp=$\frac{1}{2}$



Now by reflecting the negative part of the graph through x-axis,

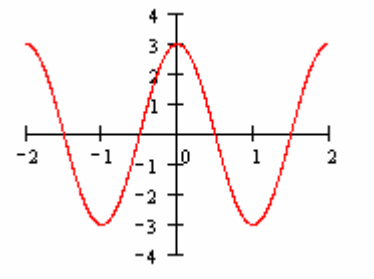
we obtained the graph $y=\left|\frac{1}{2}\sin 2x\right|$



Now the period of $y = \left|\frac{1}{2}\sin 2x\right|$ is $\frac{\pi}{2}$ and the maximum is 0.5, and the minimum is 0.

Do exr.47, and 50, page 441.

Example# 6 Find an equation of the graph.



Solution

The equation is $y = a\cos(bx)$

$amp = 3 \rightarrow a = 3$

period is the distance between two Maximum (or minimum)

$$P = \frac{2\pi}{b} = 2 \rightarrow 2\pi = 2b \rightarrow b = \pi$$

$Then \quad y = 3\cos \pi x$

Do exr.58, and 59, page 441.