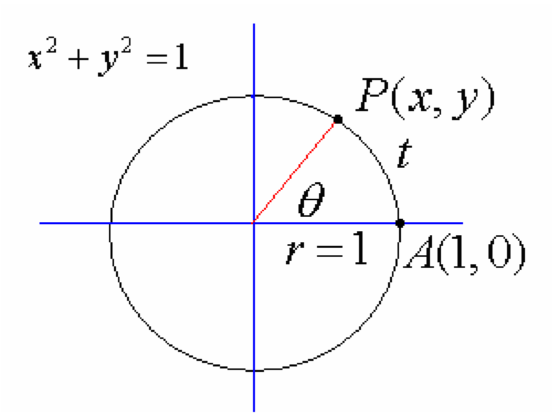
5.4 Trigonometric Functions of Real Numbers

t is arc length



Wrapping Function W(t)=p(x,y) on a unit circle.

$$s=r\theta \to t=r\theta, r=1 \to t=\theta$$

Definition of the Trigonometric Function of Real Number

Let W be the wrapping function, t be a real number, and $W(t)=P(x,y)$ be the point on the unit circle. Then

$$\sin t = y \qquad \cos t = x \qquad \tan t = \frac{y}{x}, x \neq 0$$

$$\csc t = \frac{1}{y}, y \neq 0 \quad \sec t = \frac{1}{x}, x \neq 0 \quad \cot t = \frac{x}{y}, y \neq 0$$

Notes:

$W(t)=(\cos t, \sin t)$

$\sin(-x) = -\sin x, \quad \cos(-x)=\cos x, \quad \tan(-x) = -\tan x$

$\csc(-x) = -\csc(x), \quad \sec(-x) = \sec x, \quad \cot(-x) = -\cot x$

$-1 \leq \sin x \leq 1, \qquad -1 \leq \cos \leq 1$

$\csc x \leq -1 \ \ or \ \ \csc x \geq 1, \quad \sec x \leq -1 \ \ or \ \ \sec x \geq 1$

For example: Find a) $W(\pi)$ b) $W(-\frac{5\pi}{6})$

$a)\ y=\sin\pi=0,\quad x=\cos\pi=-1$

$\rightarrow W(\pi)=P(-1,0)$

$b)\qquad \frac{5\pi}{6}\rightarrow\theta'=\frac{\pi}{6}\text{in quadrant II.}$

$$\rightarrow y=\sin\left(-\frac{5\pi}{6}\right)=-\sin(\frac{5\pi}{6})=-\sin\left(\frac{5\pi}{6}\right)=-\frac{1}{2},$$

$$x=\cos(-\frac{5\pi}{6})=\cos(\frac{5\pi}{6})=-\cos(\frac{\pi}{6})=-\frac{\sqrt{3}}{2}$$

$$\rightarrow W(\frac{\pi}{6})=P(-\frac{\sqrt{3}}{2},-\frac{1}{2})$$

Do Exr. 4, 5, page 431.

Example #1 Find the exact value of each function.

a) $\csc(-\frac{5\pi}{3})$ b) $\cos(\frac{23\pi}{6})$

Solution

a)

$\frac{5\pi}{3}\rightarrow\theta'=\frac{\pi}{3},\text{in quadrant IV}\rightarrow$

$$\csc(-\frac{5\pi}{3})=-\csc(\frac{5\pi}{3})=-\left(-\csc(\frac{\pi}{3})\right)=\frac{2}{\sqrt{3}}$$

b) H.W.

Do Exr.15, 20, and 21, page 431.

Example #2 Is $f(x)=\frac{\sin x}{x}$ even, odd, or neither?

Solution

$$f(-x)=\frac{\sin(-x)}{-x}=\frac{-\sin x}{-x}=\frac{\sin x}{x}=f(x)\rightarrow \text{f is even function}.$$

Do Exr. 35, 38, and 40 page 432.

The reciprocal identities are

$$\sin t = \frac{1}{\csc t}, \quad \cos t = \frac{1}{\sec t}, \quad \tan t = \frac{1}{\cot t}$$

Ratio identities are

$$\tan t = \frac{\sin t}{\cos t} \qquad \cot t = \frac{\cos t}{\sin t}$$

Pythagorean identities are

$$\cos^2 t + \sin^2 t = 1 \quad 1 + \tan^2 t = \sec^2 t \quad 1 + \cot^2 t = \csc^2 t$$

Example #3 Write the expression $\frac{1}{1-\sin t} + \frac{1}{1+\sin t}$ as a single term.

Solution

$$\frac{1}{1-\sin t} + \frac{1}{1+\sin t} = \frac{1+\sin t + 1 - \sin t}{(1-\sin t)(1+\sin t)} = \frac{2}{1-\sin^2 t} = \frac{2}{\cos^2 t} = 2\sec^2 t$$

Do exr. 55, 56, 59, and 64 page 432.

Example #4 Write $\sec t$ in terms of $\tan t$, $\pi < t < \frac{3\pi}{2}$.

Solution

$$1 + \tan^2 t = \sec^2 t \to \sec t = \pm\sqrt{1+\tan^2 t}.$$

Because $\pi < t < \frac{3\pi}{2}$, $\sec t$ is negative. Therefore, $\sec t = -\sqrt{1+\tan^2 t}$.

See example #6 page 430, and do exr. 65, and 68, page 432.