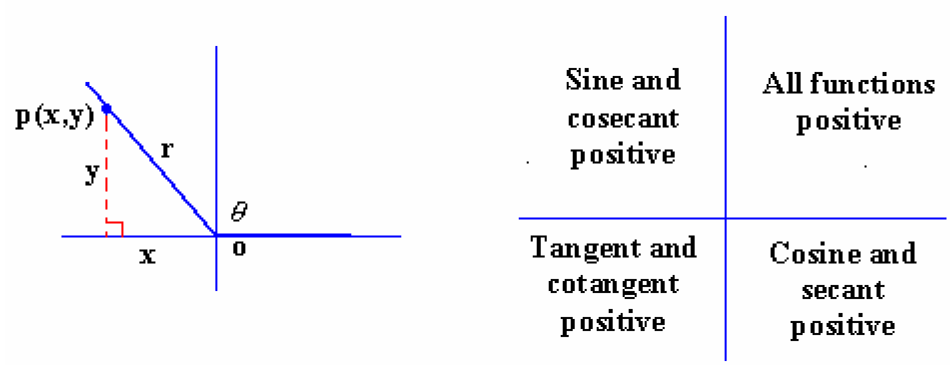
5.3 Trigonometric Functions of Any Angle



The Trigonometric Functions of Any Angle

Let P (x, y) be any point, except the origin, on the terminal side of angle θ in standard position. Let $r=\sqrt{x^2+y^2}$, the distance from the origin to P. The six trigonometric functions of angle θ are

$\sin\theta=\frac{y}{r}$ $\cos\theta=\frac{x}{r}$ $\tan\theta=\frac{y}{x},\ x\neq 0$

$\csc\theta=\frac{r}{y}, y\neq 0$ $\sec\theta=\frac{r}{x}, x\neq 0$ $\cot\theta=\frac{x}{y}, y\neq 0$

Table 6.4 Values Trigonometric Functions for Quadrantal Angles

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
0°	0	1	0	undef.	1	undef.
90°	1	0	undefined	1	undef.	0
180°	0	-1	0	undef.	-1	undef.
270°	-1	0	undefined	-1	undef.	0

Example #1 If θ is an angle whose terminal side contains the point P (-3, -1), then find the value of

a) $\tan\theta+\sin\theta$ b) $\sec\theta-\cot\theta$

Solution

a)

$$r=\sqrt{x^2+y^2}, where\, x=-3\, and\; y=-1$$
$$r=\sqrt{(-3)^2+(-1)^2}=\sqrt{10}$$
$$\sin\theta=\frac{y}{r}=\frac{-1}{\sqrt{10}}=\frac{-\sqrt{10}}{10}$$
$$\tan\theta=\frac{y}{x}=\frac{-1}{-3}=\frac{1}{3}$$
$$\tan\theta+\sin\theta=\frac{-\sqrt{10}}{10}+\frac{1}{3}=\frac{-3\sqrt{10}+10}{30}$$

b) H.W. Do exr. 4 & 8 page 421

Example #2 Find the value of each expression.

a) $\tan\theta=-\frac{1}{2}$ and $\csc\theta=\sqrt{5}$; find $\cos\theta$

b) $\cos\theta=\frac{2}{3}$ and $\sin\theta<0$; find $\tan\theta$

Solution

a)

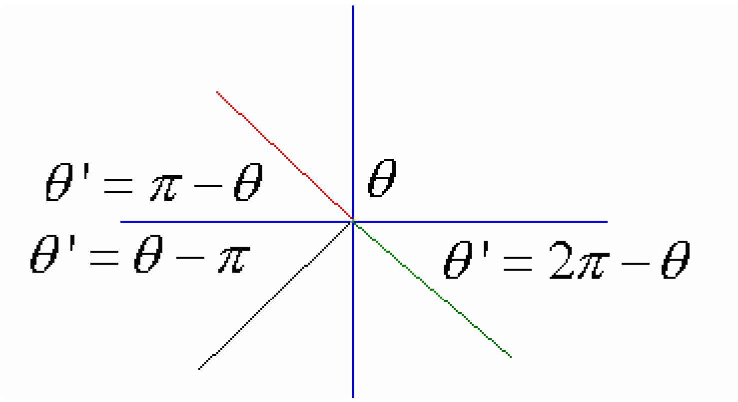
$$\tan\theta=-\frac{1}{2},\ \theta \text{ in quadrant II or IV}$$
$$\csc\theta=\sqrt{5},\ \theta \text{ in quadrant I or II}$$
$$\theta \text{ in quadrant II. } \tan\theta=-\frac{1}{2}\rightarrow\ x=-2 \text{ and } y=1$$
$$r=\sqrt{x^2+y^2}=\sqrt{1+4}=\sqrt{5}$$
$$\cos\theta=\frac{x}{r}=\frac{-2}{\sqrt{5}}=\frac{-2\sqrt{5}}{\sqrt{5}}$$

b) H.W.

Do exr. 20 and 24 page 422.

Reference Angle

Suppose θ is an angle in standard position. The reference (related) angle θ' for the angle θ is positive acute angle formed by the terminal side of θ and the x-axis.



Example #3 Find reference angle for each the following angle.

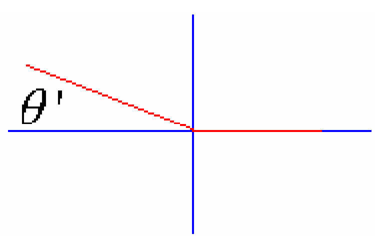
a) $\theta = 160^\circ$ b) $\beta = \dfrac{31\pi}{6}$

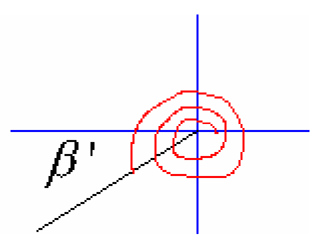
Solution

$$\theta' = \pi - 160^\circ = 20^\circ$$

$$\beta = \frac{30\pi + \pi}{6} = 5\pi + \frac{\pi}{6} = 4\pi + (\pi + \frac{\pi}{6})$$

$$\beta' = (\pi + \frac{\pi}{6}) - \pi = \frac{\pi}{6}$$





Do exr.31, 32, and 36 page 422

Reference Angle Theorem

Trigonometric functions of $\theta = \pm$ (trigonometric functions of θ'), depending on which quadrant θ.

Example #4 Find the exact value of each expression.

a) $\tan\frac{31\pi}{6} + 2\sin 315^\circ$ b) $\cot 480^\circ - \cos\frac{5\pi}{3}$ c) $\sec(-120^\circ)$

Solution

a)

$$\frac{31\pi}{6} = \frac{30\pi + \pi}{6} = 5\pi + \frac{\pi}{6} = 4\pi + \pi + \frac{\pi}{6} \equiv \pi + \frac{\pi}{6} \text{ in quadrant III}$$

$$\rightarrow \frac{31\pi}{6} \text{ in quadrant III} \rightarrow \theta' = (\pi + \frac{\pi}{6}) - \pi = \frac{\pi}{6}$$

$$\therefore \tan\frac{31\pi}{6} = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\text{since } 315^\circ \text{ in quadrant IV} \rightarrow \theta' = 360^\circ - 315^\circ = 45^\circ$$

$$\therefore \sin 315^\circ = -\sin 45^\circ = -\frac{1}{\sqrt{2}} = \frac{-\sqrt{2}}{2}$$

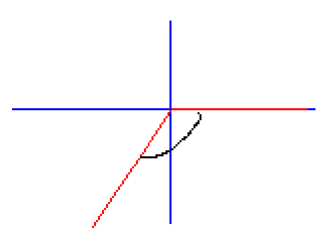
$$\therefore \tan\frac{31\pi}{6} + 2\sin 315 = \frac{\sqrt{3}}{3} + 2\frac{-\sqrt{2}}{2} = \frac{\sqrt{3}}{3} - \sqrt{2}$$

b) H.W.

c)

$$-120^\circ \text{ in quadrantIII} \rightarrow \theta' = 60^\circ$$

$$\sec(-120^\circ) = -\sec 60^\circ = -2$$



Do exr. 39, 42, 46, 61, and 66, page 422