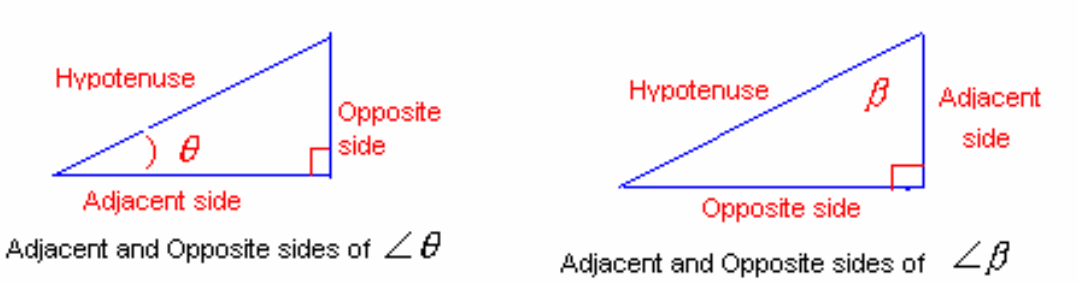
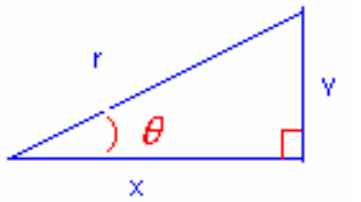
5.2 Trigonometric Functions of Acute Angles



Now consider the Figure



The six trigonometric functions of angle θ are sine(sin), cosine(cos), tangent(tan), cosecant(csc), secant(sec), and cotangent(cot).

Trigonometric Functions of Acute Angles

Let θ be an acute angle of right triangle. The values of six trigonometric functions of angle θ are

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{y}{r} \qquad \cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{x}{r}$$

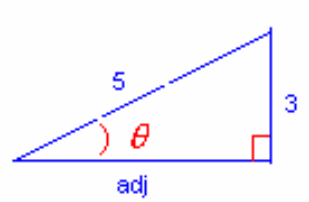
$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{y}{x} \qquad \cot\theta = \frac{\text{adjacent side}}{\text{opposite side}} = \frac{x}{y}$$

$$\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent side}} = \frac{r}{x} \qquad \csc\theta = \frac{\text{hypotenuse}}{\text{opposite side}} = \frac{r}{y}$$

Example #1 Given $\sin\theta = \frac{3}{5}$, find $\tan\theta$ and $\cos\theta$

Solution

$\sin\theta = \frac{3}{5} = \frac{opp}{hyp}$ Sketch a right triangle



Use the Pythagoreans theorem to find adj.

$5^2 = 3^2 + (adj)^2$

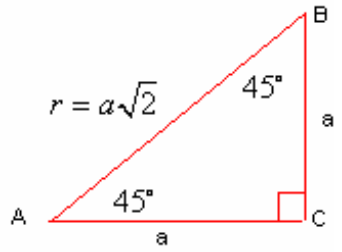
$(adj)^2 = 16 \rightarrow adj = \sqrt{16} = 4$

Therefore, $\tan\theta = \frac{opp}{adj} = \frac{3}{4}$, $\cos\theta = \frac{adj}{hyp} = \frac{4}{5}$

Do exr. 16, 18, 22, 24, pages 411

Trigonometric Functions of Special Angles

a) "$45^\circ - 45^\circ$" Right Triangle (two sides are equal)



The values of the six trigonometric functions of 45° are

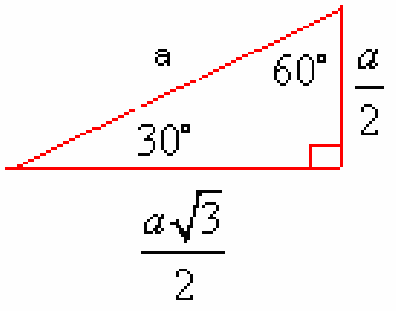
$\sin 45^\circ = \frac{a}{a\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ $\cos 45^\circ = \frac{a}{a\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\tan 45^\circ = \frac{a}{a} = 1$ $\cot 45^\circ = \frac{a}{a} = 1$

$\sec 45^\circ = \frac{a\sqrt{2}}{a} = \sqrt{2}$ $\csc 45^\circ = \frac{a\sqrt{2}}{a} = \sqrt{2}$

b) $30^\circ - 60^\circ$ Right Triangle

(*The side opposite of* 30° *is half of hypotenuse.*)



The values of the six trigonometric functions of 30° are

$$\sin 30^\circ = \frac{a/2}{a} = \frac{1}{2} \qquad \cos 30^\circ = \frac{\sqrt{3}a/2}{a} = \frac{\sqrt{3}}{2}$$

$$\tan 30^\circ = \frac{a/2}{\sqrt{3}a/2} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \qquad \cot 30^\circ = \frac{\sqrt{3}a/2}{a/2} = \sqrt{3}$$

$$\sec 30^\circ = \frac{a}{\sqrt{3}a/2} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3} \qquad \csc 30^\circ = \frac{a}{a/2} = 2$$

The values of the six trigonometric functions of 60° are

$$\sin 60^\circ = \frac{\sqrt{3}a/2}{a} = \frac{\sqrt{3}}{2} \qquad \cos 60^\circ = \frac{a/2}{a} = \frac{1}{2}$$

$$\tan 60^\circ = \frac{\sqrt{3}a/2}{a/2} = \sqrt{3} \qquad \cot 60^\circ = \frac{a/2}{\sqrt{3}a/2} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\sec 60^\circ = \frac{a}{a/2} = 2 \qquad \csc 60^\circ = \frac{a}{\sqrt{3}a/2} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

Example #2 Find the *exact* value of $\cos\frac{\pi}{4}\tan\frac{\pi}{6}+2\tan\frac{\pi}{3}$.

Solution

$$\cos\frac{\pi}{4}\tan\frac{\pi}{6}+2\tan\frac{\pi}{3}=\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{3}\right)+2\left(\sqrt{3}\right)$$

$$=\frac{\sqrt{6}}{6}+2\sqrt{3}=\frac{\sqrt{6}+12\sqrt{3}}{6}$$

Do exr. 32, 35, and 34, page 411.

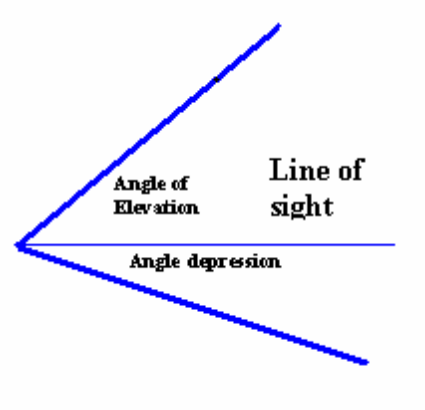
Table 6.3 Trigonometric Function and Their Reciprocals

$\sin\theta=\frac{1}{\csc\theta}$ $\csc\theta=\frac{1}{\sin\theta}$	$\cos\theta=\frac{1}{\sec\theta}$ $\sec\theta=\frac{1}{\cos\theta}$	$\tan\theta=\frac{1}{\cot\theta}$ $\cot\theta=\frac{1}{\tan\theta}$

Application of Right Triangles

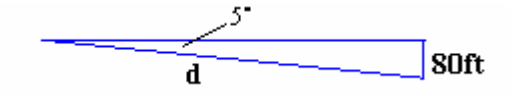
Notes:

- An angle measured above the line of sight is called angle of elevation.
- An angle measured below the line of sight is called an angle of depression.



Example #3 Submarine traveling 9.0 mph is descending at an angle of depression of 5°. How many minutes does it take the submarine to reach a depth of 80 feet?

Solution



$$\sin 5^\circ = \frac{80}{d} \rightarrow d = \frac{80}{\sin 5^\circ}\text{ft} \approx 917.9$$

$$t = \frac{d}{v},\ v = 9\frac{\text{mi}}{\text{h}} = 9\frac{\text{mi}}{\text{h}} \bullet 5280\frac{\text{ft}}{\text{mi}} \bullet \frac{1\text{h}}{60\text{min}}$$

$$= 792\text{ft/min}$$

$$t = \frac{d}{v} = \frac{917.9ft}{798\,ft/\text{min}} \approx 1.16\,\text{min}$$

Do exr. 67 and 66 page 412