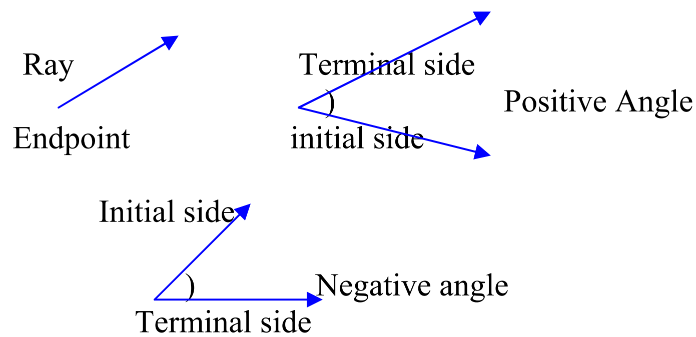
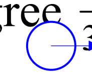
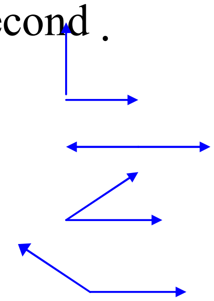
CHAPTER 5

Trigonometric Functions

5.1 Angle and Arcs



Notes:

- Positive angle, if the direction of rotation is counterclockwise.
- Negative angle, if the direction of rotation is clockwise.
- One complete revolution is 360° degree. One degree $\frac{1}{360}$ revolution, $1^\circ = 60'$ minute, $1' = 60''$ second.
  
- 90° Angles are right angles.
- 180° Angles are straight angles.
- $0^\circ < \theta < 90^\circ$ θ is acute angle.
- $90^\circ < \theta < 180^\circ$ θ is obtuse angle.
  
- Two positive angles are complementary angles if the sumof the measures of the angles is 90°.
- Two positive angles are supplementary angles if the sum of the measures of the angles is 180°.

- An angle is in standard position if its vertex at the origin and initial side lies along positive x-axis.

- Measures of Coterminal Angles
  Given $\angle\theta$ in standard position, then the measures of angles that are coterminal with θ are given by $\theta + k*360^\circ$ where k is an integer.

Example #1 Find the measure of complement and supplement of each angle.

a) $\theta = 75^\circ$ b) $\beta = 23^\circ 12' 42''$ c) $\alpha = 83^\circ 32' 52''$

Solution

a) The complementary angle of θ is $90^\circ - 75^\circ = 15^\circ$. The supplementary angle of θ is $180^\circ - 75^\circ = 105^\circ$

b) The complementary angle of β is

$$90^\circ - 23^\circ 12' 42'' = 89^\circ 59' 60'' - 23^\circ 12' 42''$$

$$\rightarrow \quad 89^\circ 59' 60''$$

$$-23^\circ 12' 42''$$

$$66^\circ 47' 18''$$

The supplementary angle of β is

$$180^\circ - 23^\circ 12' 42''$$

$$= 179^\circ 59' 60'' - 23^\circ 12' 42''$$

$$\rightarrow \quad 179^\circ 59' 60''$$

$$-23^\circ 12' 42''$$

$$156^\circ 47' 18''$$

c) H.W.
Do exr. 6, 3, and 12, page 399.

Example #2 Find first positive coterminal with the given angle.

a) $\theta = -790^\circ$ b) $\alpha = -425^\circ$ c) $\beta = 533^\circ$

Solution

a)

$$-790^\circ + 360^\circ + 360^\circ = -70^\circ$$

$$-70^\circ + 360^\circ = 290^\circ$$

Then first positive coterminal angle of θ is 290°.
Since $-790^\circ = 290 + (-3) \bullet 360^\circ$.

b) H.W.

c) $533^\circ = 360^\circ + 173^\circ$
Then first positive coterminal angle of β is 173°

Do exr. 13, 18, page 399

Example #3 Convert $62^\circ 14' 34''$ to decimal degree.

Solution

$$62^\circ 14' 34'' = 62^\circ + 14' + 34''$$

$$= 62^\circ + 14'\left(\frac{1}{60}\right) + 34''\left(\frac{1}{3600}\right)$$

$$= 62^\circ + 0.23 + 0.0094 = 62.2394^\circ$$

Central Angle

An angle has its vertex at the center of a circle is called central angle.

Radian Measure

Given an arc of length s on a circle of radius r, the measure of central angle is $\theta = \frac{s}{r}$ radians

Notes: (Radian-Degree Conversion)

- Circumference of a circle with radius r is $C = 2\pi r$, then $\theta = \frac{s}{r} = \frac{2\pi r}{r} = 2\pi$ radians. ($360^\circ = 2\pi$ radian)
- To change degrees to radians, multiply by $\frac{\pi}{180}$.
- To change radians to degrees, multiply by $\frac{180}{\pi}$.

Example #4 Convert $\frac{-3\pi}{5}$ radians to degree.

Solution

$$\frac{-3\pi}{5}\text{radians} = \frac{-3\pi}{5}\left(\frac{180^\circ}{\pi}\right) = -108^\circ$$

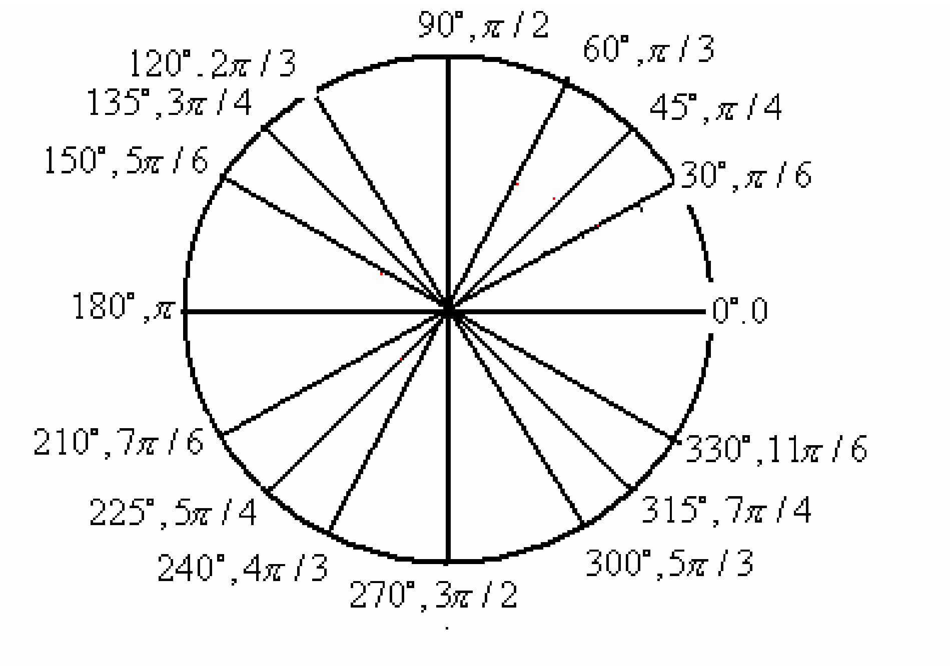
Do exr. 44, 48, page 399.

Example #5 Convert 270° to radian.

Solution

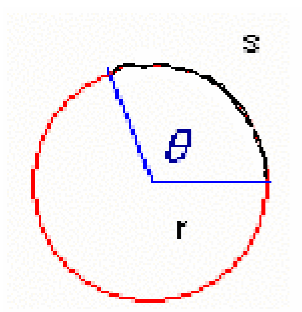
$$270^\circ = 270^\circ\left(\frac{\pi}{180}\right) = \frac{3\pi}{2}\text{radians.}$$

Do exr. 34, 36, page 399.



Arcs and Arc length

Let θ be nonnegative radian measure of a central angle of the circle with radius r. Then the length of the arc s is $s = r\theta$.



Example #6 Find the length of an arc that a central angle of 60° in a circle of radius 8 cm.

Solution

The formula $s = r\theta$ requires that θ be in radian. We first convert 60° to radian and then use the formula.

$$\theta = 60^\circ = 60^\circ\left(\frac{\pi}{180}\right) = \frac{\pi}{3}$$

$$s = r\theta = 8\left(\frac{\pi}{3}\right) = \frac{8\pi}{3}\text{cm.}$$

Linear and Angular Speed

Linear speed $v = \dfrac{\text{distance}}{\text{time}} = \dfrac{s}{t}$

Angular speed $\omega = \dfrac{\theta}{t}$**,** where θ in radian, t is the time.

$$s = r\theta \to v = \frac{s}{t} = \frac{r\theta}{t} = \omega r \to v = r\omega$$

Example #7 A wheel is rotating at 50 revolutions per minute. Find the angular speed in radian per second.

Solution

$$\omega = 50\,\text{revolution per minute}$$

$$= \left(\frac{50(2\pi)}{60}\right) = \frac{5\pi}{3}\text{radian/sec.}$$

Do Exr. 70, 72 page 400.