4.5 Exponential and Logarithmic Equations

Exponential Equations

Note: Remember that $b^x = b^y \to x = y$, for $b > 0, b \neq 1$.

Example #1 Solve for x

a) $(16)^{2x+1} = \frac{1}{2}$ b) $\frac{81}{16} = \left(\frac{2}{3}\right)^{3x-1}$ c) $3^{x+2} = 5^{2x-1}$

Solution

a)

$$(16)^{2x+1} = \frac{1}{2} \to \left(2^4\right)^{2x+1} = \frac{1}{2} \to$$

$$2^{4(2x+1)} = \frac{1}{2} \to 2^{8x+4} = \frac{1}{2} = 2^{-1}$$

$$\to 2^{8x+4} = 2^{-1} \to 8x + 4 = -1 \to x = \frac{-5}{8}$$

b) H.W.

c)

$$3^{x+2} = 5^{2x-1}, \text{take ln both sides } \to \ln 3^{x+2} = \ln 5^{2x-1}$$
$$\to (x+2)\ln 3 = (2x-1)\ln 5$$
$$\to x\ln 3 + 2\ln 3 = 2x\ln 5 - \ln 5$$
$$\to x\ln 3 - 2x\ln 5 = -\ln 5 - 2\ln 3$$
$$\to (\ln 3 - 2\ln 5)x = -(\ln 5 + 2\ln 3)$$
$$x = \frac{-(\ln 5 + \ln 9)}{\ln 3 - 2\ln 5} = \frac{-\ln 45}{\ln 3 - \ln 25}$$

Do Ex. 5, 7, 16, and 19

Logarithmic Equations

Notes:

- Remember that
  $\log_b x = \log_b y \;\rightarrow\; x = y,\;$ for $x > 0$ and $y > 0$
- Write both sides of the logarithmic equation as single logarithm.
- Check the solutions in the original equation (domain).

Example #2 Find the solution set.

a) $\log_2 x - \log_2(x-1) = 3$

b) $\ln(4-x) = \ln(8+x) + \ln(2x+13)$

Solution

a)

$$\log_2 x - \log_2(x-1) = 3 \rightarrow \log_2 \frac{x}{x-1} = 3$$ *Quotient property

$$2^3 = \frac{x}{x-1} \rightarrow 8(x-1) = x \rightarrow 8x - 8 = x \rightarrow 7x = 8$$

$$x = \frac{8}{7}$$

Solution set is $\left\{\frac{8}{7}\right\}$,

since $\frac{8}{7}$ in the domain of $\log_2 x$ and $\log_2(x-1)$.

b)

$$\ln(4-x)=\ln(8+x)+\ln(2x+13)$$
$$\ln(4-x)=\ln(8+x)(2x+13) \quad \text{*Product property}$$
$$4-x=(8+x)(2x+13)$$
$$4-x=2x^2+29x+104$$
$$2x^2+30x+100=0$$
$$x^2+15x+50=0$$
$$(x+5)(x+10)=0$$
$$x=-5 \quad or \quad x=-10$$

Solution set is $\{-5\}$ since –10 is not in the domain $\ln(8+x)$

Do Ex.30, 28, 34 and 35

Example #3 Solve.

a) $\dfrac{e^x+e^{-x}}{e^x-e^{-x}}=3$ b) $\log(\log x)=1$ c) $\ln x^2-\ln 4=\ln x$

Note: To solve an equation includes b^x *and* b^{-x}, multiply each side by b^x to clear negative exponents. Then write in quadratic form.

Solution

a)

$$\frac{e^x+e^{-x}}{e^x-e^{-x}}=3, \text{ Multiply each side by } e^x$$

$$e^x\left(\frac{e^x+e^{-x}}{e^x-e^{-x}}\right)=3e^x \rightarrow \frac{e^{2x}+e^0}{e^x-e^{-x}}=3e^x \rightarrow e^{2x}+1=3e^x(e^x-e^{-x})$$

$e^{2x}+1=3e^{2x}-3 \rightarrow 4=2e^{2x} \rightarrow 2=e^{2x}$, take ln each side

$$\ln 2=\ln e^{2x} \rightarrow \ln 2=2x\ln e \rightarrow \ln 2=2x$$

$$x=\frac{\ln 2}{2}$$

b)

$$\log(\log x)=1 \rightarrow 10^1=\log x \rightarrow 10=\log x \rightarrow 10^{10}=x$$

$$x=10^{10}$$

c) $\ln x^2-\ln 4=\ln x \rightarrow 2\ln x-\ln 4=\ln x \rightarrow \ln x=\ln 4 \rightarrow x=4$

Do Ex.32, 31, 37, and 42

More Exercises

1. Find the value of

   a) $(\log_5 16)(\log_2\sqrt{5})-(\sqrt{e})^{-6\ln 2}$

   b) $\ln\ln e^{e^{x+3}}-e^{\ln x}$

   c) $\log_8\frac{\sqrt[3]{16}}{4}$

2. Solve .

   a) $\ln(x-2)-\log_{e^{-1}}(x+2)-\ln 12=0$.

b) $\log_2 \sqrt{x-2} + \log_4(x-4) = \frac{3}{2} + \frac{1}{2}\log_2 3$.

3. If the amount of a certain radioactive material present after t days is $P(t) = 800e^{-t\ln 2}$ grams, fine the time needed for the material to decay to 200 grams.

4. In the formula $f(x) = x_0 e^{kx}$, *if* $f(25) = \frac{1}{2}x_0$, find $f(75)$.