4.4 Properties of Logarithms

Properties of Logarithms

If b, M, and N are positive real numbers, $b \neq 1$, and r is any real number, then

$$\left.\begin{cases} 1.\ \log_b b = 1 \\ 2.\ \log_b 1 = 0 \\ 3.\ \log_b b^r = r \end{cases}\right\} basic\ \ properties$$

$$4.\ \log_b MN = \log_b M + \log_b N \qquad \text{Pr}\,oduct\ property$$

$$5.\ \log_b \frac{M}{N} = \log_b M - \log_b N \qquad Quotient\ property$$

$$6.\ \log_b M^r = r \log_b M \qquad Power\ property$$

$$7.\ \log_b M = \log_b N \rightarrow M = N \qquad One-to-one\ property$$

$$8.\ b^{\log_b N} = N \qquad Inverse\ property$$

Note: $\log_b (M+N) \neq \log_b M + \log_b N$

Example#1 Write each logarithmic expression as a single logarithm.

a) $2\log_3 x^2 y - 3\log_3 x^2$

b) $\log_{\frac{1}{2}}(x^2 - y^2) + \frac{1}{3}\log_{\frac{1}{2}} x^3 y^6 - 2\log_{\frac{1}{2}}(x-y)$

Solution

a)

$$2\log_3 x^2y - 3\log_3 x^2 = \log_3(x^2y)^2 - \log_3(x^2)^3$$

$$= \log_3 \frac{(x^2y)^2}{(x^2)^3} = \log_3 \frac{x^4y^2}{x^6} = \log_3 \frac{y^2}{x^2}$$

b) H.W. Do Ex. 14 &18 page 339

Example#2 Evaluate each logarithm

a) $\log_2 32$ b) $\log .001$ c) $\ln e^5$ d) $\log_{\frac{2}{3}} \frac{9}{4}$

Solution

a) $\log_2 32 = \log_2 2^5 = 5\log_2 2 = 5$

b) $\log .001 = \log \frac{1}{1000} = \log 1 - \log 1000$

$$= 0 - \log 10^3 = -3\log 10 = -3$$

c) $\ln e^5 = 5\ln e = 5$ d) H.W.

Example#3 If $\log 3 = x$ and $\log 2 = y$, express the following logarithm in terms of x and y.

a) $\log \frac{81}{4}$ b) $\log 45$ c) $\log 75$

Solution

a) $\log \frac{81}{4} = \log 81 - \log 4 = \log 3^4 - \log 2^2$.

$$= 4\log 3 - 2\log 2 = 4x - 2y$$

b) $$\log 45 = \log(5)(9) = \log 5 + \log 9 = \log\frac{10}{2} + \log 3^2$$
$$= \log 10 - \log 2 + 2\log 3 = 1 - y + 2x$$

c) H.W.

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Example#4 Find all real number that are solution of the given inequality.

a) $-3 \le \log_2(x+3) \le 1$ b) $1 \le \ln(x+1) < 3$

c) $\log_3 x \le 2$ d) $12 - 2\log_2(x+3) > 0$

e) $4 - \ln(2-x) \ge 0$.

Solution

a) $-3 \le \log_2(x+3) \le 1$

$$2^{-3} \le 2^{\log_2(x+3)} \le 2^1$$

$$\to \frac{1}{8} \le x+3 \le 2$$

$$\to \frac{1}{8} - 3 \le x \le 2 - 3 \to \frac{-23}{8} \le x \le -1$$

Since the domain of $\log_2(x+3)$ *is* $(-3,\infty)$

$$\frac{-23}{8} \le x \le -1 \bigcap (-3,\infty)$$

$$\to \frac{-23}{8} \le x \le -1$$

b) H.W. $ans\ is\ e-1 \le x < e^3 - 1$

c)

$$\log_3 x \le 2$$

$$3^{\log_3 x} \le 3^2$$

$$\rightarrow\ x \le 9$$

Since the domain of $\log_3 x$ is $(0,\infty)$

$$\rightarrow x \le 9 \cap (0,\infty)$$

$$\rightarrow 0 < x \le 9$$

d)

$$12 - 2\log_2(x+3) > 0\ \rightarrow\ 12 > 2\log_2(x+3)$$

$$\rightarrow 6 > \log_2(x+3)$$

$$2^6 > 2^{\log_2(x+3)}$$

$$\rightarrow 64 > x+3\quad \rightarrow 61 > x$$

Since the domain of $\log_2(x+3)$ is $(-3,\infty)$

$$x < 61 \cap (-3,\infty) \rightarrow$$

$$-3 < x < 61$$

e) H.W $ans\ \ is\ \ 2 - e^4 \le x < 2$

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H.W. If g is logarithmic function, and g(64)=3. Find g(1/2).

Change - of - Base Formula

If x, a and b are positive real number with $a \neq 1$ *and* $b \neq 1$, then

$$\log_b x = \frac{\log_a x}{\log_a b}$$

Example#5 Evaluate $\log_3 5 \cdot \log_5 7 \cdot \log_7 9$

Solution

$$\log_3 5 \cdot \log_5 7 \cdot \log_7 9 = \log_3 5 \cdot \frac{\log_3 7}{\log_3 5} \cdot \frac{\log_3 9}{\log_3 7} = \log_3 9$$

$$= \log_3 3^2 = 2$$

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