4.3 Logarithmic Function and Their Graphs

Definition of a Logarithm

If $x > 0$ and b is a positive real number, $(b \neq 1)$, then

$$y = \log_b x \quad \textit{if and only if} \quad b^y = x,$$

y is called **logarithm**, b is the base, and x is the argument.

The **logarithmic function with base b** is defined by

$$f(x) = \log_b x,$$

Note:

- The logarithmic function $f(x) = \log_b x$ is the inverse the exponential function $g(x) = b^x$.
- $y = \log_b x$ is the **logarithmic form**, while $b^y = x$ is the **exponential form.**

Example#1 Write each equation in its exponential form.

a) $3 = \log_{\frac{1}{2}} x$ b) $2 = \log_5(x+1)$ c) $-5 = \log_{10}(x-2)$

Solution

a) $3 = \log_{\frac{1}{2}} x \quad \rightarrow \quad \left(\frac{1}{2}\right)^3 = x$

b) $2 = \log_5(x+1) \quad \rightarrow \quad 5^2 = x+1$

c) H.W.

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Example#2 Write each equation in its logarithmic form.

a) $y = 5^{-9}$ b) $x+1 = \left(\frac{1}{3}\right)^4$

Solution

a) $y = 5^{-9} \rightarrow -9 = \log_5 y$

b) $x+1 = \left(\frac{1}{3}\right)^4 \rightarrow 4 = \log_{\frac{1}{3}}(x+1)$

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Basic Logarithmic Properties

1. $\log_b b = 1$ 2. $\log_b 1 = 0$ 3. $\log_b b^m = m$

Example#3 Evaluate each of the following:

$a.\ \log_6 36$ $b.\ \log_3 81$ $c.\ \log_2 \frac{1}{4}$

Solution

$a.\ \log_6 36 = \log_6 6^2 = 2$

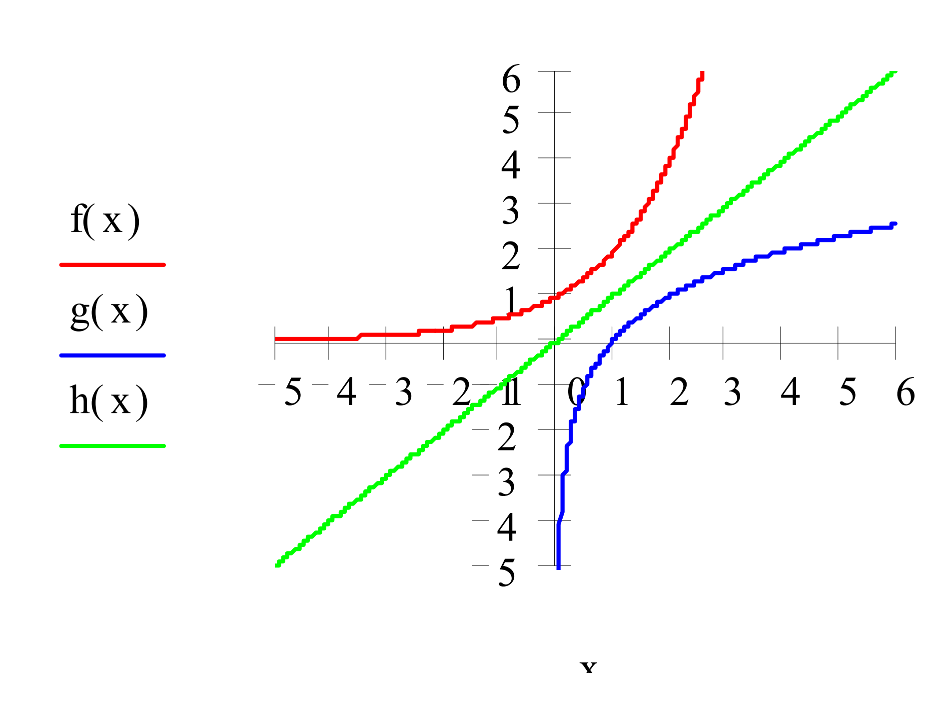
$b.\ \log_3 81 = \log_3 3^4 = 4$

$c.\ \log_2 \frac{1}{4} = \log_2 4^{-1} = \log_2 2^{-2} = -2$

Notes: The graph of $y = \log_b x$ can be obtained by reflecting the graph of $y = b^x$ across the line $y = x$.

$f(x) = 2^x$

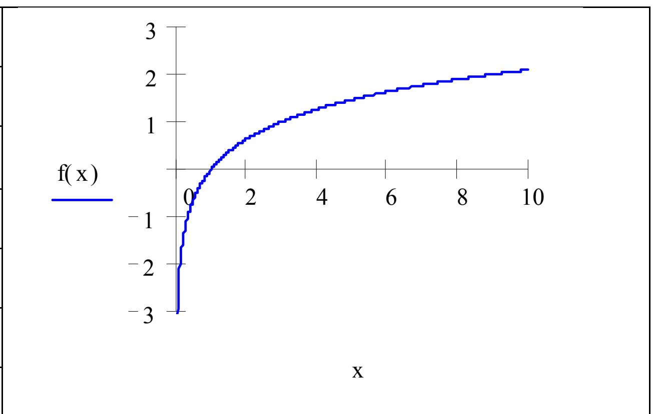
$g(x) = \log_2 x$



Example #4 Logarithmic Function with Base b > 1

$$f(x) = \log_3 x$$

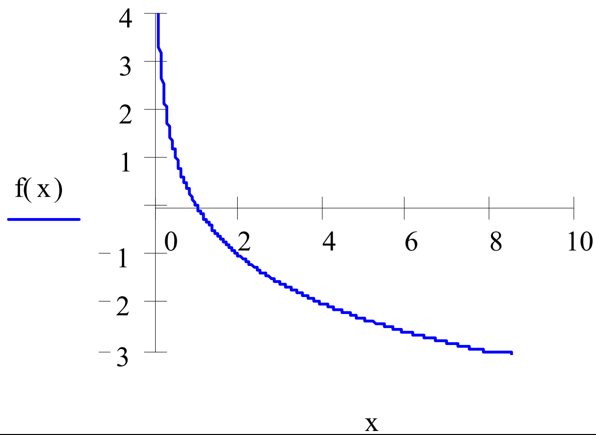
x	F(x)
1/27	-3
1/9	-2
1/3	-1
1	0
3	1
9	2



Example #5 Logarithmic Function with Base 0 < b < 1

$$f(x) = \log_{\frac{1}{2}} x$$

x	f(x)
1/8	3
1/4	2
1/2	1
1	0
2	1
4	2



Properties of $f(x) = \log_b x$

For b>0 and $b \neq 1$, $f(x) = \log_b x$ has the following properties:

1. Domain of f is all positive real numbers, $(0, \infty)$.
2. Range of f is all real numbers, $(-\infty, \infty)$.
3. The graph of f has x-intercept (1,0).
4. y-axis is a V.A. of the graph f.
5. f is one-to-one function.
6. f is increasing function if b>1.
7. f is decreasing function if 0<b<1.

Common and Natural Logarithm

Logarithms with base of 10 are called **common logarithm**

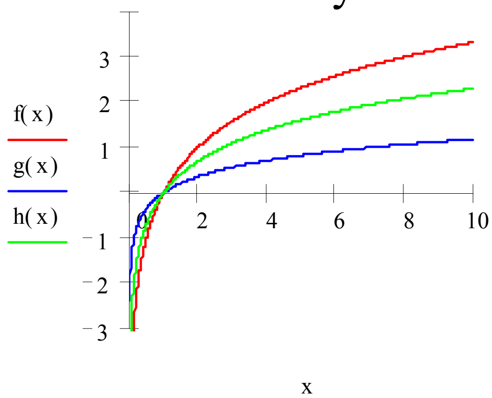
$$\log_{10} x = \log x$$

Logarithms with base of e are called **natural logarithm**

$$\log_e x = \ln x.$$

Example #6 Sketch the graphs of

$f(x)=\log_2 x,\quad g(x)=\log_6 x,$ and $h(x)=\ln x$ on the same coordinates system.



Example #7 Graph

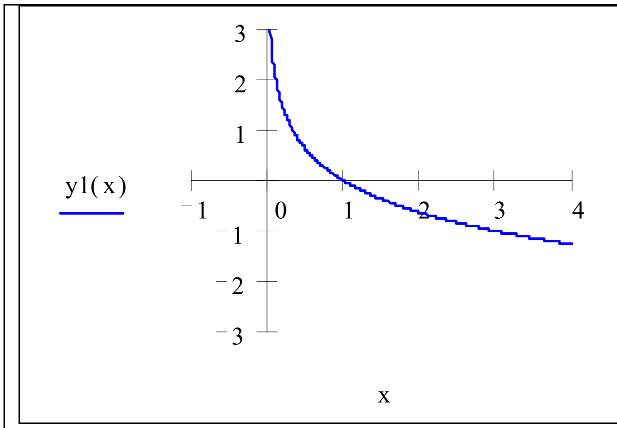
a) $f(x)=-\log_{\frac{1}{3}}(x-4)$ b) $g(x)=\ln(-x)+5$

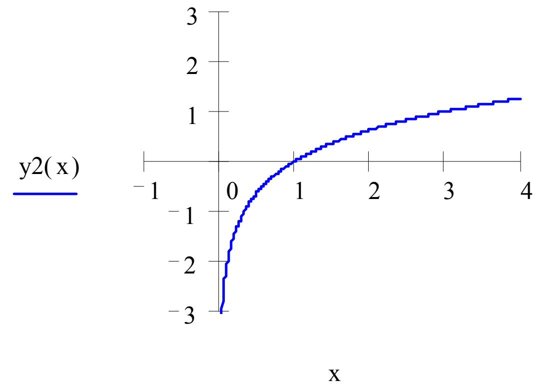
c) $h(x)=-\log(-x+3)-4$

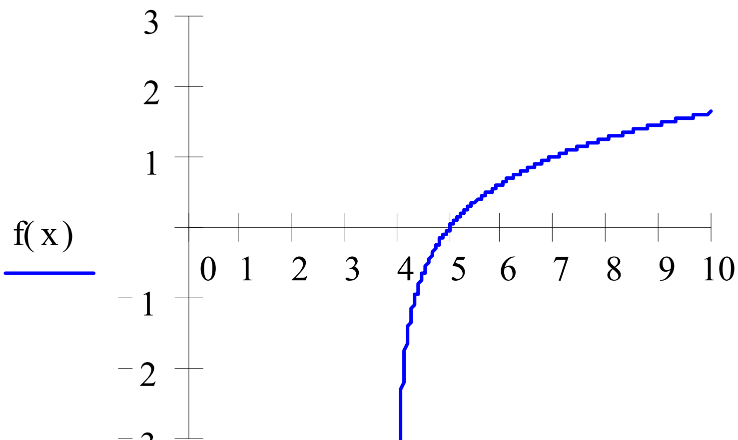
Solution

a)

First we graph $y1=\log_{\frac{1}{3}} x$, reflecting the graph through x-axis we obtain the graph of $y2=-\log_{\frac{1}{3}} x$, Shifting the graph 4 units to the right we obtain the graph of $f(x)$







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b) and c) H.W.

Example #8 Graph

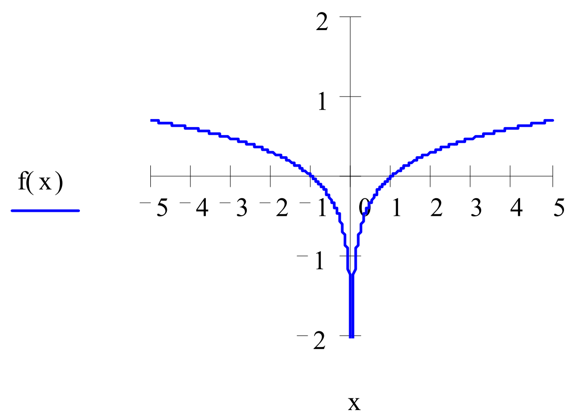
a) $f(x) = \log|x|$ b) $g(x) = \ln|x-4|$

Solution

$$f(-x) = \log|-x| = \log|x|, \rightarrow$$

a) the graph of f is symmetric w.r.t . y-axis,

take $x > 0 \rightarrow |x| = x \rightarrow f(x) = \log x$



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b) H.W.

Example #9 Find the domain and V.A. of

$f(x) = -\ln(3x-5)+2$

Solution

Domain: argument greater than zero

$$3x-5>0 \;\rightarrow\; 3x>5 \;\rightarrow\; x>\frac{5}{3} \;\rightarrow D=\left(\frac{5}{3}\ ,\ \infty\right)$$

V.A. argument greater equal zero

$$3x-5=0 \rightarrow\; x=\frac{5}{3}$$