

CHAPTER 4

Exponential and Logarithmic Functions

4.2 Exponential Functions and Their Graphs

Definition of an exponential function

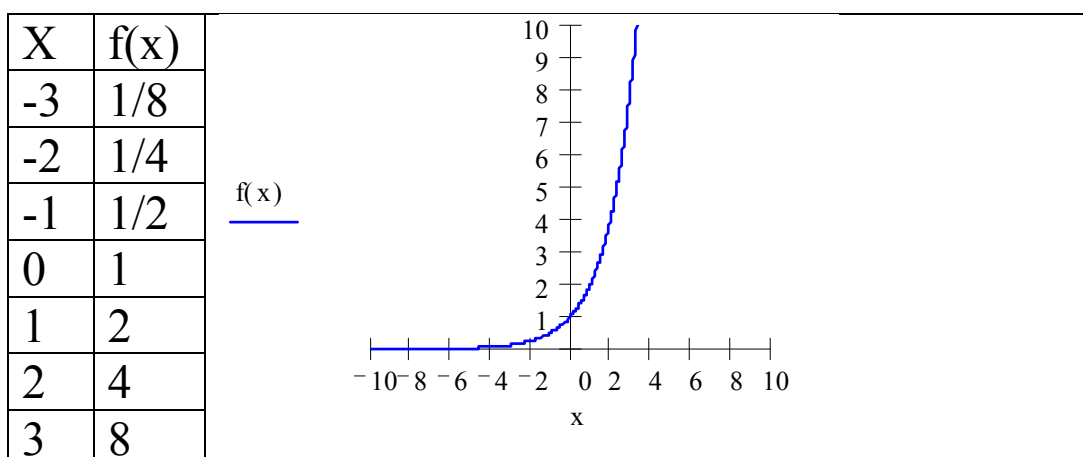
If b is a positive real number and $b \neq 1$, the function f defined by

$$f(x) = b^x$$

is an **exponential function with base b** .

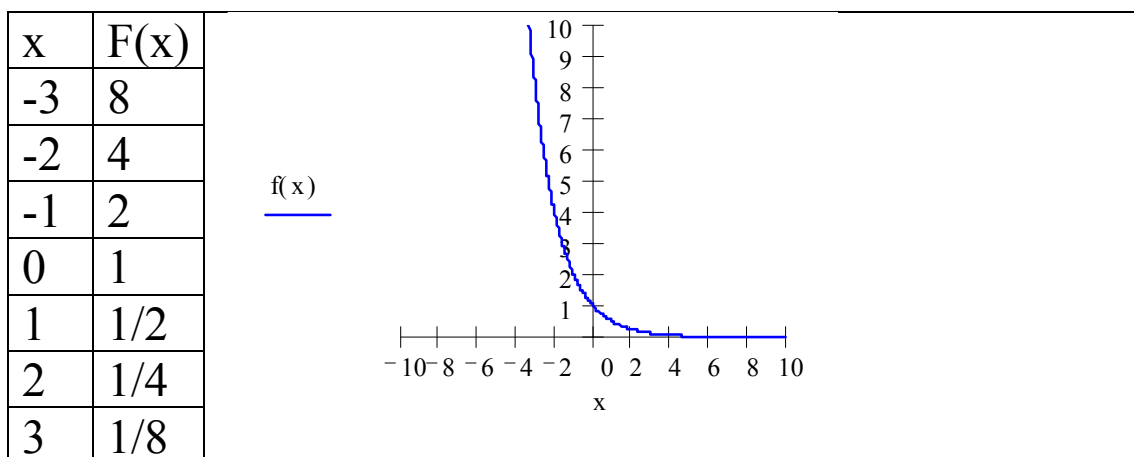
Example #1 Exponential Function with Base $b > 1$.

$$f(x) = 2^x$$



Example #2 Exponential Function with Base $0 < b < 1$.

$$f(x) = \left(\frac{1}{2}\right)^x$$



Properties of $f(x) = b^x$

For $b > 0$ and $b \neq 1$, $f(x) = b^x$ has the following properties:

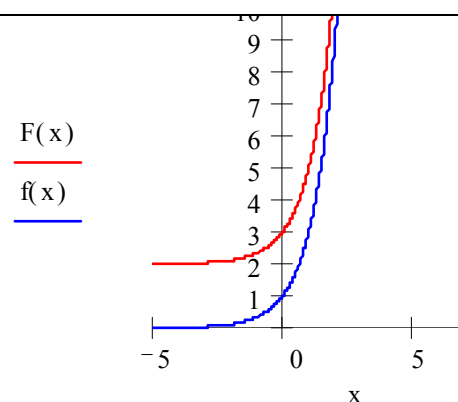
1. Domain of f is all real numbers, $(-\infty, \infty)$.
2. Range of f is all positive real numbers, $(0, \infty)$.
3. The graph of f has y-intercept $(0, 1)$.
4. x-axis is a H.A. of the graph f .
5. f is one-to-one function.
6. f is increasing function if $b > 1$.
7. f is decreasing function if $0 < b < 1$.

Example #3 Sketch the graph of each function.

a) $F(x) = 3^x + 2$ b) $G(x) = \left(\frac{1}{3}\right)^{x-2}$

a)

The graph of
 $F(x) = 3^x + 2$
 is the graph of
 $f(x) = 3^x$
 Shifted up 2 units



b)

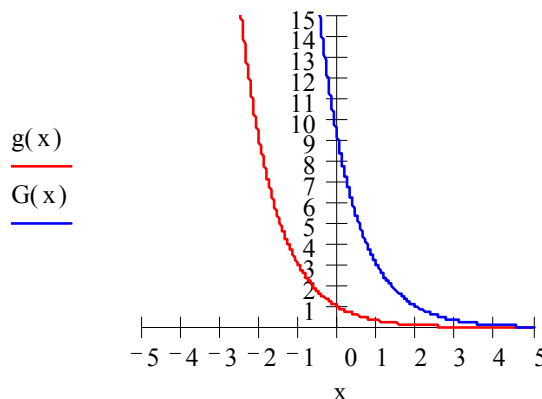
The graph of

$$G(x) = \left(\frac{1}{3}\right)^{x-2}$$

is the graph of

$$g(x) = \left(\frac{1}{3}\right)^x$$

Shifted 2 units to the right



Do Exercise 38, 49, and 50 page 317

The Natural Exponential Function.

For all real numbers x , the function defined by

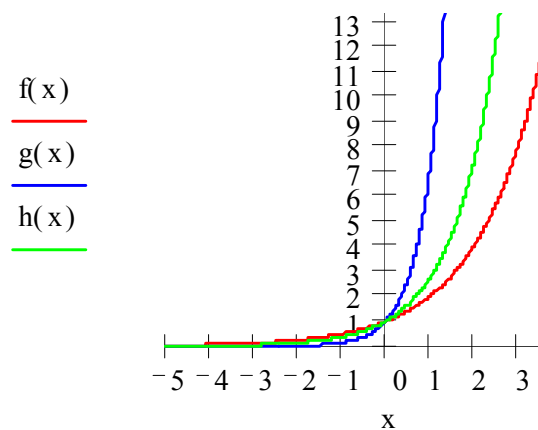
$$f(x) = e^x$$

is called the **natural exponential function.**

Note: $\left(1 + \frac{1}{n}\right)^n \rightarrow e \approx 2.71828$ as $n \rightarrow \infty$.

Example #4 Sketch the graphs of

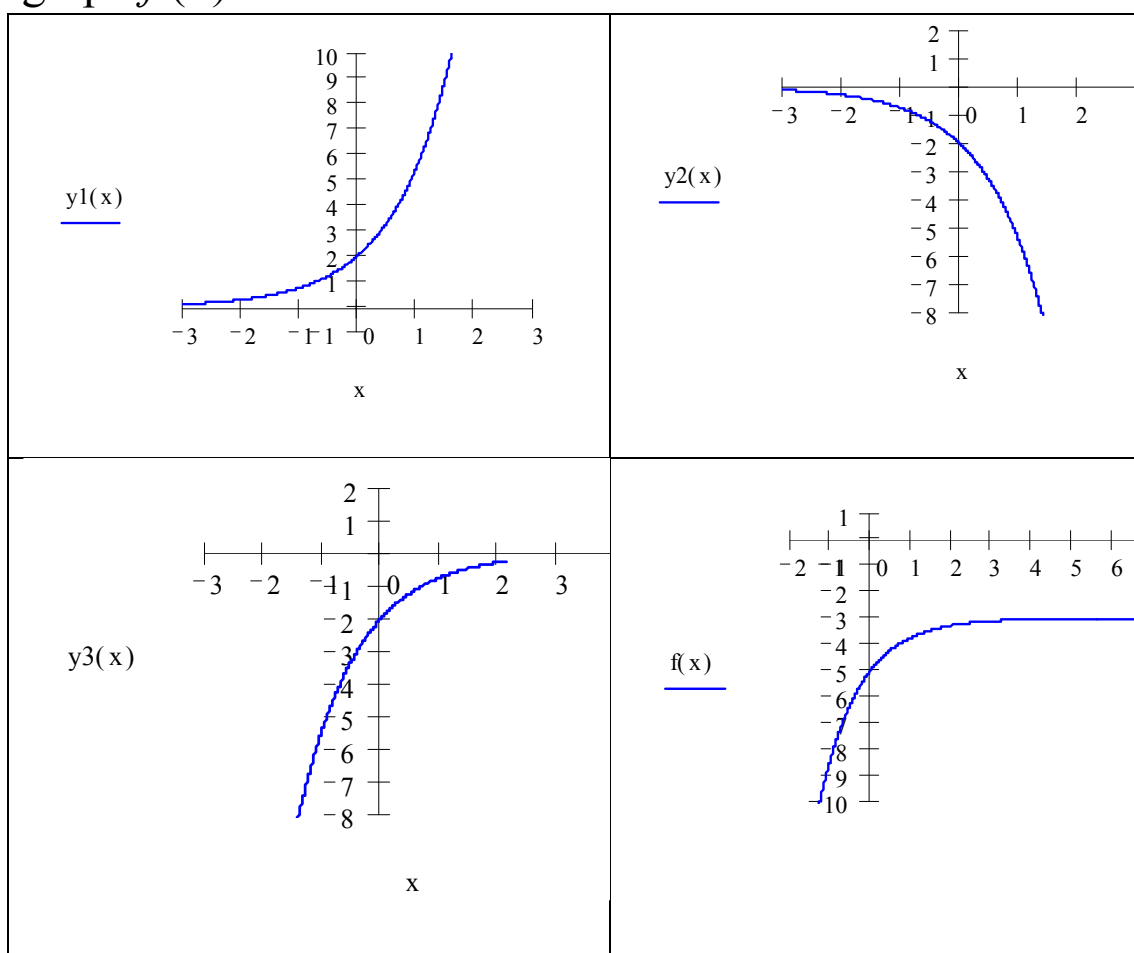
$f(x) = 2^x$, $g(x) = 7^x$, and $h(x) = e^x$ on the same Coordinates system.



Example #5 Sketch the graph of $f(x) = -2e^{-x} - 3$

Solution

First we graph $y_1 = 2e^x$ reflected through x-axis, then graph $y_2 = -2e^x$ reflected through y-axis, then graph $y_3 = -2e^{-x}$ shifted down 3 units, then graph $f(x) = -2e^{-x} - 3$.

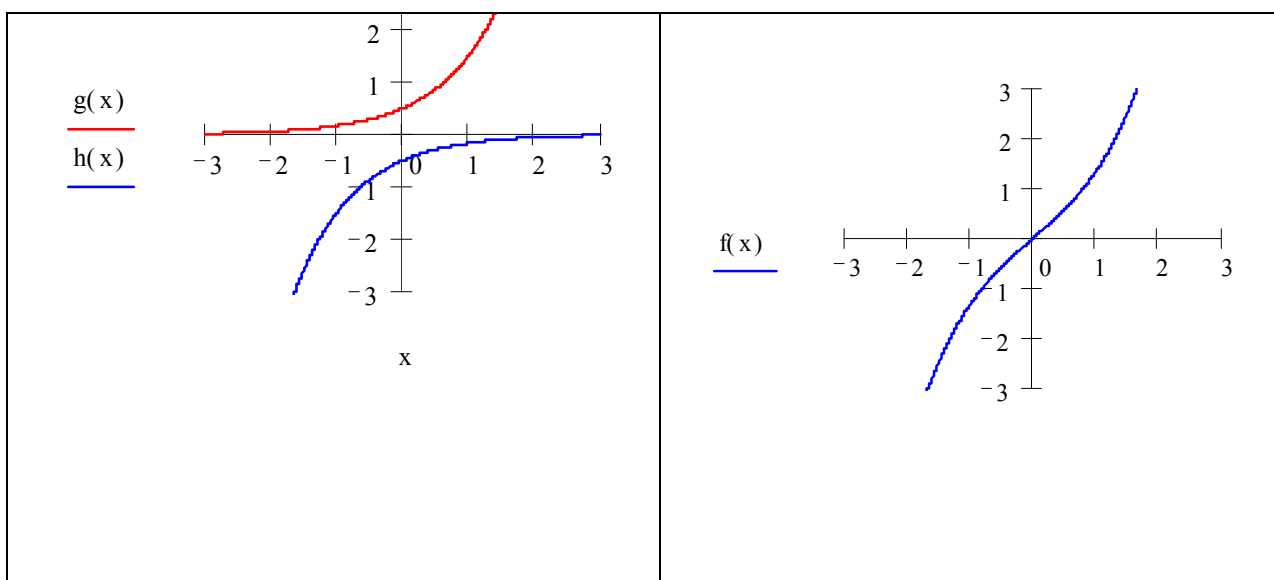


H.W Sketch the graph $f(x) = e^{-|x|}$. See exercise 40 page 279

Example #6 Sketch the graph of $f(x) = \frac{3^x - 3^{-x}}{2}$.

Solution

Let $g(x) = \frac{3^x}{2}$ and $h(x) = -\frac{3^{-x}}{2}$, then $f(x) = g(x) + h(x)$



Do Exercise 41 and 48 page 317

H.W. If g is an exponential function, and $g(2)=4$. Find $g(5)$.