King Fahd University of Petroleum & Minerals
Department of Mathematical Sciences
Math 102 Quiz # 1

Name: _____ I.D. # _____ Section # 01

1. Evaluate the integral $\int \frac{dx}{x(\ln x^2)^2}$, $x > 0$

Let $u = \ln x^2 \rightarrow du = \frac{1}{x^2} \cdot 2x\,dx = \frac{2}{x}dx$

$$I = \int \frac{1}{x\,u^2} \cdot \frac{x\,du}{2} = \frac{1}{2}\int u^{-2}du = \frac{1}{2}\frac{u^{-1}}{-1} + c =$$

$$= -\frac{1}{2}\frac{1}{u} + c = -\frac{1}{2}\frac{1}{\ln x^2} + c$$

2. Evaluate the integral $\int \frac{e^{3x} - e^x}{e^x + 1}dx$ $= I$

Let $u = e^x \rightarrow du = e^x dx = u\,dx$

$$I = \int \frac{u^3 - u}{u+1} \cdot \frac{du}{u} = \int \frac{u(u-1)(u+1)}{(u+1)u}du = \frac{u^2}{2} - u + c$$

$$= \frac{e^{2x}}{2} - e^x + c$$

3. Use the rectangular method ( with the right endpoints) to find the net signed area between the graph $f(x) = x^2 - x$ and x-axis over $[-1, 1]$.

$\Delta x = \frac{1+1}{n} = \frac{2}{n}$, $x_k^* = -1 + k\frac{2}{n} = \frac{-n+2k}{n}$

$$A = \lim_{n\to\infty} \sum_{k=1}^{n} \left[\left(\frac{-n+2k}{n}\right)^2 - \left(\frac{-n+2k}{n}\right)\right]\frac{2}{n} = \lim_{n\to\infty} \frac{2}{n}\sum_{k=1}^{n}\left[\frac{n^2 - 4kn + 4k^2}{n^2} + 1 - \frac{2k}{n}\right]$$

$$= \lim_{n\to\infty} \frac{2}{n}\sum_{k=1}^{n}\left[1 - \frac{4k}{n} + \frac{4k^2}{n^2} + 1 - \frac{2k}{n}\right] = \lim_{n\to\infty} \frac{2}{n}\left[2n - \frac{6}{n}\sum_{k=1}^{n} k + \frac{4}{n^2}\sum_{k=1}^{n} k^2\right]$$

$$= \lim_{n\to\infty} \frac{2}{n}\left[2n - \frac{6}{n}\frac{n(n+1)}{2} + \frac{4}{n^2}\frac{(n)(n+1)(2n+1)}{6}\right]$$

$$= \lim_{n\to\infty} 2\left[2 - 3\frac{n+1}{n} + \frac{4}{6}\frac{(n+1)(2n+1)}{n^2}\right]$$

$$= \lim_{n\to\infty} 2\left[2 - 3 + \frac{2}{3}\cdot 2\right] = 2\left[-1 + \frac{4}{3}\right] = 2\left[\frac{1}{3}\right] = \frac{2}{3}$$