10.3 The Inverse Matrix

Multiplicative Inverse of a Matrix

Is A is a square matrix of order n, then the **inverse** of matrix A, denoted by A^{-1}, has the property that

$$A \cdot A^{-1} = A^{-1} \cdot A = I_n$$

where I_n is the identity matrix of order n.

Procedure for Finding A^{-1}

1. Write A with identity matrix of the same order: $\left[A \vdots I_n\right]$
2. Use row operations to transform $\left[A \vdots I_n\right]$ into the form $\left[I_n \vdots B\right]$. If this is possible, then $B = A^{-1}$. If this is not possible, then A^{-1} does not exist. A is called **singular** matrix.

Example #1 Find the inverse of each matrix.

a) $\begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & 3 \\ 2 & 1 & 3 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

Solving System of Equations Using Inverse matrix

Consider the system of equations

$$-2x+3y=-2$$
$$x+2y=\ \ 1$$

This system is equivalent to the matrix equation $AX=B$, where

$$A=\begin{bmatrix}-2 & 3\\ 1 & 2\end{bmatrix},\qquad X=\begin{bmatrix}x\\ y\end{bmatrix},\quad B=\begin{bmatrix}-2\\ 1\end{bmatrix}$$

Then

$$X=A^{-1}B=\begin{bmatrix}-\frac{2}{7} & \frac{3}{7}\\ \frac{1}{7} & \frac{2}{7}\end{bmatrix}\cdot\begin{bmatrix}-2\\ 1\end{bmatrix}$$

$$X=\begin{bmatrix}\frac{4}{7}+\frac{3}{7}\\ -\frac{2}{7}+\frac{2}{7}\end{bmatrix}=\begin{bmatrix}1\\ 0\end{bmatrix}$$

Thus $x=1,\ \ y=0$. The solution is $(1,0)$

Example #2 Solve the system of equations

$$x+2y-\ \ z=5$$
$$2x+3y-\ \ z=8$$
$$3x+6y-2z=14$$

by using inverse matrix method.