10.1 Gaussian Elimination Method

Matrices are useful tools in solving system of linear equations. In this section we consider one of the best-known matrix method, the **Gaussian elimination method**.

A matrix can be created from a system of linear equations. Consider the system of linear equations

$$\begin{aligned} 3x+2y-3z &= -1 \\ x+3y+2z &= 1 \\ x+y-2z &= -3 \end{aligned}$$

Using only the coefficients and constants of this system, we can write the matrices:

$$A=\begin{bmatrix} 3 & 2 & -3 \\ 1 & 3 & 2 \\ 1 & 1 & -3 \end{bmatrix},\quad B=\begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, and\ \left[A\vdots B\right]=\begin{bmatrix} 3 & 2 & -3 & \vdots & -1 \\ 1 & 3 & 2 & \vdots & 1 \\ 1 & 1 & -3 & \vdots & -3 \end{bmatrix}$$

A is called coefficient matrix of the system; B is called the constant matrix of the system; $\left[A\vdots B\right]$ is called augmented matrix of the system.

Gaussian Elimination Method

1. Write the augmented matrix $\left[A\vdots B\right]$.
2. Use any of **row operations**:
   a. Interchange two rows.
   b. Multiply (or divide) every element in a row by the same nonzero number.
   c. Add (or subtract) a multiple of one row to (or from) another row.

To $\left[A \vdots B\right]$ in **echelon form**; all elements on the main diagonal are 1's and below it are 0's.

3. Use back substitution on the system that has the augmented obtained in step 2.

Example #1 Use Gaussian elimination to solve the following system

a)

$$\begin{aligned} 3x+2y-3z &= -1 \\ x+3y+2z &= 1 \\ x+y-2z &= -3 \end{aligned}$$

Ans. (3, -2, 2)

b)

$$\begin{aligned} 2x+5y+2z &= -1 \\ x+2y-3z &= 5 \\ 5x+12y+z &= 10 \end{aligned}$$

Ans. no solution

c)

$$\begin{aligned} 3x-5y+2z &= 4 \\ x-3y+2z &= 4 \\ 5x-11y+6z &= 12 \end{aligned}$$

Ans. (c-2, c-2, c)