King Fahd University of Petroleum and Minerals
Math Prep-Year Program Math 002
Term 051 **Quiz# 5**

Name: ______________________ **ID#:**__________________ **Sec.09---**

Q1 Find the equation in standard form oh the hyperbola that has foci at (-2, 1) and (-4, 1). slope of the asymptote $\frac{1}{2}$.

Horizontal $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

center$(h,k) = \left(\frac{-2+(-4)}{2}, \frac{1+1}{2}\right) = (-3,1)$

$\boxed{c = 1}$ $1 = a^2+b^2$ --- (1)

slope of asymptote $\frac{b}{a} = \frac{1}{2} \rightarrow \boxed{a = 2b}$ -- (2)

substitute in (1) $\rightarrow 1 = 4b^2+b^2 \rightarrow b^2 = \frac{1}{5}$

substitute in (2) $\rightarrow a^2 = 4b^2 = 4\left(\frac{1}{5}\right)$

$a^2 = \frac{4}{5}$

$$\frac{(x+3)^2}{\frac{4}{5}} - \frac{(y-1)^2}{\frac{1}{5}} = 1$$

Q2. Solve for x and y $(4-3i)x + (3+2i)y = 7+8i$

$4x - 3xi + 3y + 2yi = 7+8i$

$(4x+3y) + (-3x+2y)i = 7+8i$

$4x+3y = 7$ and $-3x+2y = 8$

-2 / $4x+3y = 7$ --- (1)

3 / $-3x+2y = 8$ --- (2)

$-17x + 0 = 10$

$x = -\frac{10}{17}$

substitute in (2)

$-3\left(\frac{10}{-17}\right) + 2y = 8 \rightarrow \frac{30}{17} + 2y = 8 \rightarrow 2y = 8 - \frac{30}{17} = \frac{106}{17}$

$y = \frac{53}{17}$

Q3. Given the system of equations

$$x + 3y - a^2 z = a^2$$
$$2x + 3y + az = 2$$
$$3x + 4y + 2z = 3$$

Find all values of a for which the system of the equation has a unique solution.

$$\left[\begin{array}{ccc|c} 1 & 3 & -a^2 & a^2 \\ 2 & 3 & a & 2 \\ 3 & 4 & 2 & 3 \end{array}\right] \xrightarrow[-3R_1+R_3]{-2R_1+R_2} \left[\begin{array}{ccc|c} 1 & 3 & -a^2 & a^2 \\ 0 & \boxed{-3} & 2a^2+a & -2a^2+2 \\ 0 & -5 & 3a^2+2 & -3a^2+3 \end{array}\right]$$

$$-\frac{1}{3}R_2 \left[\begin{array}{ccc|c} 1 & 3 & -a^2 & a^2 \\ 0 & 1 & \frac{-2a^2-a}{3} & \frac{2a^2-2}{3} \\ 0 & -5 & 3a^2+2 & -3a^2+3 \end{array}\right] \xrightarrow{5R_2+R_3} \left[\begin{array}{ccc|c} 1 & 3 & -a^2 & a^2 \\ 0 & 1 & \frac{-2a^2-a}{3} & \frac{2a^2-2}{3} \\ 0 & 0 & \frac{-10a^2-5a}{3}+3a^2+2 & \frac{10a^2-10}{3}-3a^2+3 \end{array}\right]$$

~~no~~ unique solution $\rightarrow \frac{-10a^2-5a}{3} + 3a^2 + 2 \neq 0$

$$-10a^2 - 5a + 9a^2 + 6 \neq 0$$
$$-a^2 - 5a + 6 \neq 0 \rightarrow a^2 + 5a - 6 \neq 0$$
$$(a+6)(a-1) \neq 0 \qquad a \neq -6 \quad a \neq -1$$

Q4. If $A = \begin{bmatrix} 2 & -1 & 3 & 4 \\ -2 & 0 & 1 & -3 \\ 2 & 0 & 5 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 & 5 & -1 & 0 \\ -1 & 1 & 0 & 3 & 1 \\ -5 & 0 & -2 & 3 & 0 \\ 2 & 1 & 0 & 3 & 0 \end{bmatrix}$ and $C = AB$, find C_{31}

$$C_{31} = 4 + 0 - 25 + 2 = -19$$