King Fahd University of Petroleum and Minerals
Math Prep-Year Program Math 002
Term 051 Quiz# 5

Name: ______________________ **ID#:**__________________ **Sec.05---**

Q1 Find the equation in standard form oh the hyperbola that has foci at (-2, 1) and (-4, 1), slope of the asymptote 2.

Horizontal $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Center $(h,k) = \left(\frac{-2+(-4)}{2}, \frac{1+1}{2}\right) = (-3, 1)$

$\boxed{c = 1} \rightarrow 1 = a^2 + b^2 \quad \cdots (1)$

slope of asymptote $\frac{b}{a} = 2 \rightarrow \boxed{b = 2a} \cdots (2)$

substitute $b = 2a$ in eq. (1)

$\rightarrow 1 = a^2 + 4a^2 \rightarrow \boxed{a^2 = \frac{1}{5}}$

substitute in eq. (2)

$b^2 = 4a^2 = 4\left(\frac{1}{5}\right) \rightarrow b^2 = \frac{4}{5}$

$$\frac{(x+3)^2}{\frac{1}{5}} - \frac{(y-1)^2}{\frac{4}{5}} = 1$$

Q2. Solve for x and y $\quad (4-3i)x + (5+2i)y = 12+8i$

$4x - 3xi + 5y + 2yi = 12 + 8i \Rightarrow (4x+5y) + (-3x+2y)i = 12 + 8i$

$4x + 5y = 12$ and $-3x + 2y = 8$

$2 / \; 4x + 5y = 12 \quad \cdots (1)$

$-5 / \; -3x + 2y = 8 \quad \cdots (2)$

$23x + 0 = \cancel{8} -16$

$\boxed{x = \frac{-16}{23}}$

substitute in eq (2) $\rightarrow -3\left(\frac{-16}{23}\right) + 2y = 8 \rightarrow \frac{48}{23} + 2y = 8$

$2y = 8 - \frac{48}{23} = \frac{126}{23} \rightarrow y = \frac{63}{23}$

Q3. Given the system of equations

$$x + 3y - a^2 z = a^2$$
$$2x + 3y + az = 2$$
$$3x + 4y + 2z = 3$$

Find all values of a for which the system of the equation has infinitely many solutions.

$$\left[\begin{array}{ccc|c} 1 & 3 & -a^2 & a^2 \\ 2 & 3 & a & 2 \\ 3 & 4 & 2 & 3 \end{array}\right] \xrightarrow[-3R_1+R_3]{-2R_1+R_2} \left[\begin{array}{ccc|c} 1 & 3 & -a^2 & a^2 \\ 0 & \boxed{-3} & 2a^2+a & -2a^2+2 \\ 0 & -5 & 3a^2+2 & -3a^2+3 \end{array}\right]$$

$$-\frac{1}{3}R_2 \left[\begin{array}{ccc|c} 1 & 3 & -a^2 & a^2 \\ 0 & 1 & \frac{-2a^2-a}{3} & \frac{2a^2-2}{3} \\ 0 & -5 & 3a^2+2 & -3a^2+3 \end{array}\right] \xrightarrow{5R_2+R_3} \left[\begin{array}{ccc|c} 1 & 3 & -a^2 & a^2 \\ 0 & 1 & \frac{-2a^2-a}{3} & \frac{2a^2-2}{3} \\ 0 & 0 & \frac{-10a^2-5a}{3}+3a^2+2 & \frac{10a^2-10}{3}-3a^2+3 \end{array}\right]$$

Infinitely many $\quad \frac{-10a^2-5a}{3}+3a^2+2=0$ and $\frac{10a^2-10}{3}-3a^2+3=0$

$-10a^2-5a+9a^2+6=0$ and $10a^2-10-9a^2+9=0$

$-a^2-5a+6=0$ $\qquad a^2-1=0$

$a^2+5a-6=0$ $\qquad a=\pm 1$

$(a+6)(a-1)=0$

$a=-6$ or $a=1$ and $a=1,\ a=-1$

$$\boxed{a=1}$$

Q4. If $A=\begin{bmatrix} 2 & -1 & 3 & 4 \\ -2 & 0 & 1 & -3 \\ 2 & 0 & 5 & 1 \end{bmatrix}$, $B=\begin{bmatrix} 2 & 3 & 5 & -1 & 0 \\ -1 & 1 & 0 & 3 & 1 \\ -5 & 0 & -2 & 3 & 0 \\ 2 & 1 & 0 & 3 & 0 \end{bmatrix}$ and $C=AB$, find C_{32}

$C_{32}=6+0+0+1=7$