

Q1 Find the equation in standard form of the hyperbola that has foci at (1, -2) and (1, -4), slope of the asymptote 2.

Vertical $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

Center = $\left(\frac{1+1}{2}, \frac{-2+(-4)}{2}\right) = (1, -3)$

$c = 1$

$c^2 = a^2 + b^2 \rightarrow 1 = a^2 + b^2$ (1)

slope of asymptote $\frac{a}{b} = 2 \rightarrow a = 2b$ (2)

Substitute $a = 2b$ in eq (1)

$1 = 4b^2 + b^2 \rightarrow b^2 = \frac{1}{5}$

From eq (2) $a^2 = 4b^2 = 4 \cdot \frac{1}{5}$

$a^2 = \frac{4}{5}$

$$\frac{(y+3)^2}{\frac{4}{5}} - \frac{(x-1)^2}{\frac{1}{5}} = 1$$

Q2. Solve for x and y $(4-3i)x + (5+2i)y = 11+9i$

$4x - 3xi + 5y + 2yi = 11 + 9i \Rightarrow (4x + 5y) + (-3x + 2y)i = 11 + 9i$

$\Rightarrow 4x + 5y = 11$ and $-3x + 2y = 9$

$3 \begin{array}{l} 4x + 5y = 11 \\ -3x + 2y = 9 \end{array}$

$4 \begin{array}{l} 4x + 5y = 11 \\ -3x + 2y = 9 \end{array}$

$0 + 23y = 69$

$y = 3$

$\Rightarrow 4x + 5(3) = 11 \Rightarrow 4x = -4 \Rightarrow x = -1$

Q3. Given the system of equations

$$x + 3y - a^2z = a^2$$

$$2x + 3y + az = 2$$

$$3x + 4y + 2z = 3$$

Find all values of a for which the system of the equation has no solutions.

$$\left[\begin{array}{ccc|c} 1 & 3 & -a^2 & a^2 \\ \textcircled{2} & 3 & a & 2 \\ \textcircled{3} & 4 & 2 & 3 \end{array} \right] \xrightarrow{\substack{-2R_1+R_2 \\ -3R_1+R_3}} \left[\begin{array}{ccc|c} 1 & 3 & -a^2 & a^2 \\ 0 & -3 & 2a^2+a & -2a^2+2 \\ 0 & -5 & 3a^2+2 & -3a^2+3 \end{array} \right]$$

$$\xrightarrow{-\frac{1}{3}R_2} \left[\begin{array}{ccc|c} 1 & 3 & -a^2 & a^2 \\ 0 & 1 & \frac{-2a^2-a}{3} & \frac{2a^2-2}{3} \\ 0 & -5 & 3a^2+2 & -3a^2+3 \end{array} \right] \xrightarrow{=R_1+R_2} \left[\begin{array}{ccc|c} 1 & 3 & -a^2 & a^2 \\ 0 & 1 & \frac{-2a^2-a}{3} & \frac{2a^2-2}{3} \\ 0 & 0 & \frac{-10a^2-5a}{3} + 3a^2+2 & \frac{10a^2-10}{3} + -3a^2+3 \end{array} \right]$$

No solution $\Rightarrow \frac{-10a^2-5a}{3} + 3a^2+2 = 0$ and $\frac{10a^2-10}{3} - 3a^2+3 \neq 0$

$$\Rightarrow -10a^2 - 5a + 9a^2 + 6 = 0$$

$$-a^2 - 5a + 6 = 0$$

$$a^2 + 5a - 6 = 0$$

$$(a+6)(a-1) = 0$$

$$a = -6 \text{ or } a = 1$$

$$\text{and } 10a^2 - 10 - 9a^2 + 9 \neq 0$$

$$a^2 - 1 \neq 0$$

$$(a-1)(a+1) \neq 0$$

$$a \neq 1 \text{ and } a \neq -1$$

$$\text{and } a \neq 1 \text{ or } a \neq -1$$

$$\therefore \boxed{a = -6}$$

Q4. If $A = \begin{bmatrix} 2 & -1 & 3 & 4 \\ -2 & 0 & 1 & -3 \\ 2 & 0 & 5 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 & 5 & -1 \\ -1 & 1 & 0 & 3 \\ -5 & 0 & -2 & 3 \\ 2 & 1 & 0 & 3 \end{bmatrix}$ and $C = AB$, find C_{24}

$$C_{24} = 2 + 0 + 3 - 9 = -4$$