8.5 Integrating Rational Functions by Partial Fractions

$\dfrac{P(x)}{Q(x)}$ is a proper rational function [degree of *P* < degree of *Q*]

$$\frac{P(x)}{Q(x)} = F_1(x) + F_2(x) + \cdots + F_n(x)$$

$F_1, F_2, \cdots, F_n$, are of the form $\dfrac{A}{(ax+b)^m}, \dfrac{Ax+B}{\left(ax^2+bx+c\right)^m}$

LINEAR FACTOR RULE. For each factor of the form $(ax+b)^m$, the partial fraction decomposition contains the following sum of m partial fractions:

$$\frac{A_1}{ax+b} + \frac{A_2}{(ax+b)^2} + \cdots + \frac{A_m}{(ax+b)^m}$$

where $A_1, A_2, \ldots, A_m$ are constants to be determined. In the case where $m = 1$, only the first term in the sum appears.

Example 1 Evaluate

$$a)\int \frac{dx}{x^2+x+2} \qquad b)\int \frac{2x^2+x-2}{(x+1)(x-2)^2}dx$$

QUADRATIC FACTOR RULE. For each factor of the form $(ax^2+bx+c)^m$, the partial fraction decomposition contains the following sum of m partial fractions:

$$\frac{A_1x+B_1}{ax^2+bx+c}+\frac{A_2x+B_2}{(ax^2+bx+c)^2}+\cdots+\frac{A_mx+B_m}{(ax^2+bx+c)^m}$$

where $A_1, A_2, \ldots, A_m, B_1, B_2, \ldots, B_m$ are constants to be determined. In the case where $m=1$, only the first term in the sum appears.

Example 2 Evaluate

$$a)\int \frac{x^2-x-21}{2x^3-x^2+8x-4}dx \qquad b)\int \frac{5x^3-3x^2+7x-3}{\left(x^2+1\right)^2}dx$$

$\frac{P(x)}{Q(x)}$ is an improper rational function [degree of *P* > degree of *Q*]

Example 3 Evaluate $\int \frac{3x^4 + 3x^3 - 5x^2 + x - 1}{x^2 + x - 2} dx$