8.4 Trigonometric Substitutions

Expression	Substitution	θ
$\sqrt{a^2 - x^2}$	$x = a\sin\theta$	$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$
$\sqrt{a^2 + x^2}$	$x = a\tan\theta$	$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$
$\sqrt{x^2 - a^2}$	$x = a\sec\theta$	$0 \le \theta < \frac{\pi}{2}$ if $x \ge a$ $\frac{\pi}{2} < \theta \le \pi$ if $x \le -a$

Example 1 Evaluate

$$a)\int \frac{dx}{x^2\sqrt{16 - x^2}} \qquad b)\int \frac{dx}{\sqrt{4 + x^2}} \qquad c)\int_3^6 \frac{\sqrt{x^2 - 9}}{x}dx$$

Example 2 Evaluate

$$a)\int \sqrt{5-4x-x^2}\,dx$$

$$b)\int \frac{\left(1-x^2\right)^{3/2}}{x^6}\,dx$$

$$c)\int \frac{x}{x^2-4x+8}\,dx$$