8.3 Trigonometric Integrals

$$\int \sin^n x \, dx = -\frac{1}{n}\sin^{n-1} x \cos x + \frac{n-1}{n}\int \sin^{n-2} x \, dx$$

$$\int \cos^n x \, dx = \frac{1}{n}\cos^{n-1} x \sin x + \frac{n-1}{n}\int \cos^{n-2} x \, dx$$

Example 1 Evaluate $\int \sin^4 x \, dx$

$$\int \tan^n x \, dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x \, dx$$

$$\int \sec^n x \, dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x \, dx$$

Example 2 Evaluate $\int \sec^5 3x \, dx$

Integrating Products of Sine and Cosine

$\int \sin^m x \cos^n x \, dx$	Procedure	Relevant Identities
n odd	• Split off a factor of ***cos x***. • Apply the relevant identity. •Make the substitution ***u=sin x.***	$\cos^2 x = 1 - \sin^2 x$
m odd	• Split off a factor of ***sin x***. • Apply the relevant identity. •Make the substitution ***u=cosx .***	$\sin^2 x = 1 - \cos^2 x$
$\begin{cases} m \text{ even} \\ n \text{ even} \end{cases}$	• Use the relevant identities to reduce the powers on *sinx* *and cosx*	$\begin{cases} \sin^2 x = \frac{1}{2}(1 - \cos 2x) \\ \cos^2 x = \frac{1}{2}(1 + \cos 2x) \end{cases}$

Example 3 Evaluate

$a) \int \sin^5 x \cos^4 x \, dx$ $\qquad$ $b) \int \sin^4 x \cos^6 x \, dx$

Integrating Products of Sine and Cosine

$\int \tan^m x \sec^n x \, dx$	Procedure	Relevant Identities
n even	• Split off a factor of $\sec^2 x$ • Apply the relevant identity. •Make the substitution ***u=tan x.***	$\sec^2 x = \tan^2 x + 1$
m odd	• Split off a factor of ***secx tanx*** • Apply the relevant identity. •Make the substitution ***u=sec x.***	$\tan^2 x = \sec^2 x - 1$
$\begin{cases} m \text{ even} \\ n \text{ odd} \end{cases}$	• Use the relevant identities to reduce the integrand to powers of ***secx*** alone. • Then use the reduction formula for ***secx***	$\tan^2 x = \sec^2 x - 1$

Example 4 Evaluate

$a)\ \int \tan^3 x \sec^5 x\, dx \qquad b)\ \int \tan^3 x \sec^4 x\, dx \qquad c)\ \int \tan^4 x \sec^3 x\, dx$

Example 5 Evaluate $\int \cot^4 x \csc^3 x\, dx$

The forms $\int \sin mx \cos nx dx$, $\int \sin mx \sin nx dx$, $\int \cos mx \cos nx dx$

Use the identities

$$\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha-\beta)+\sin(\alpha+\beta)\right]$$

$$\sin\alpha\sin\beta = \frac{1}{2}\left[\cos(\alpha-\beta)-\cos(\alpha+\beta)\right]$$

$$\cos\alpha\cos\beta = \frac{1}{2}\left[\cos(\alpha-\beta)+\cos(\alpha+\beta)\right]$$

Example 6 Evaluate $\int \sin 3x \cos 4x\, dx$