Principles of Integral Evaluation

8.2 Integration by Parts

$$d(uv) = udv + vdu \quad \Rightarrow \quad \int d(uv) = \int (udv + vdu) \quad \Rightarrow$$

$$uv = \int udv + \int vdu \quad \Rightarrow$$

$$\int udv = uv - \int vdu$$

The application of this formula is called **integration by parts.**

To evaluate $\int f(x)g(x)dx$, *where* $G'(x) = g(x)$

$$u = f(x) \qquad dv = g(x)dx$$

$$du = f'(x)dx \qquad v = \int g(x)dx = G(x)$$

$$Then \int f(x)g(x)dx = \int udv = uv - \int vdu$$

Example 1 Evaluate the integrals

$a)\ \int x e^{2x}\,dx$ $\qquad b)\ \int x^2 \sin x\,dx$ $\qquad c)\ \int_0^{\pi/3} x \sec^2 x\,dx$

Example 2 Evaluate $\int e^x \cos x\,dx$

Example 3 Evaluate the integrals

$a)\ \int \tan^{-1} x\,dx$ $\qquad b)\ \int \ln x\,dx$ $\qquad HW$ $\qquad c)\ \int \cos(\ln x)\,dx$

Example 4 Evaluate $\int \sec x\ dx$

Reduction Formulas

$$\int \sin^n x \, dx = -\frac{1}{n}\sin^{n-1} x \cos x + \frac{n-1}{n}\int \sin^{n-2} x \, dx$$

$$\int \cos^n x \, dx = \frac{1}{n}\cos^{n-1} x \sin x + \frac{n-1}{n}\int \cos^{n-2} x \, dx$$

Example 5 Evaluate $\int \sin^5 x \, dx$