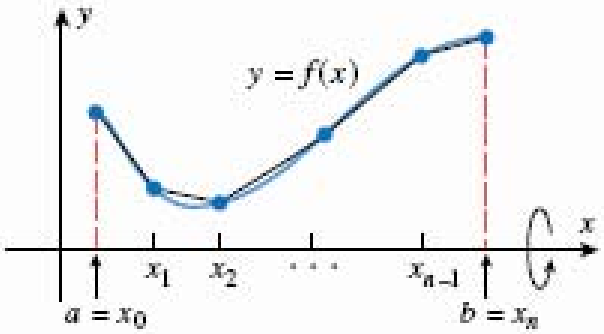
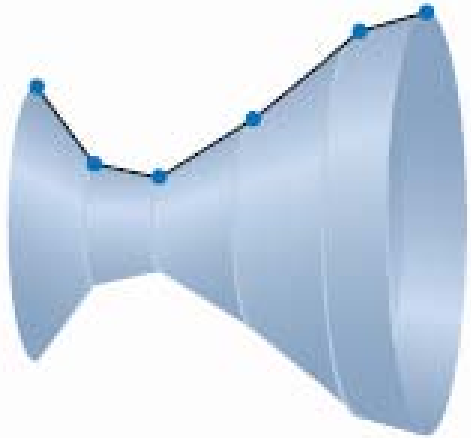
7.5 Area of a Surface of Revolution





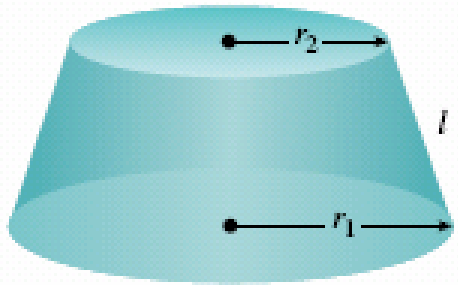
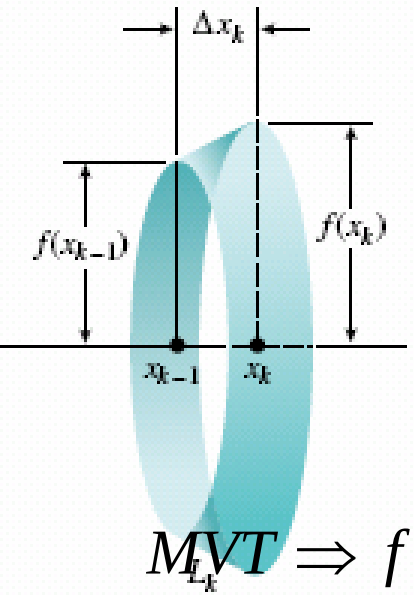


Figure 7.5.4



$$S = \pi (r_1 + r_2) l$$

$$L_k = \sqrt{(\Delta x_k)^2 + [f(x_k) - f(x_{k-1})]^2}$$

$$S_k = \pi [f(x_k) + f(x_{k-1})] \sqrt{(\Delta x_k)^2 + [f(x_k) - f(x_{k-1})]^2}$$

$$MVT \Rightarrow f'(x_k^*) = \frac{f(x_k) - f(x_{k-1})}{x_k - x_{k-1}} \quad or \quad f(x_k) - f(x_{k-1}) = f'(x_k^*)\Delta x_k$$

$$S = \lim_{\max \Delta x_k \to 0} \sum_{k=1}^{n} 2\pi \left[\frac{f(x_k) + f(x_{k-1})}{2} \right] \sqrt{1 + [f'(x_k^*)]^2}\, \Delta x_k$$

$$S = \lim_{\max \Delta x_k \to 0} \sum_{k=1}^{n} 2\pi f(x_k^{**}) \sqrt{1 + \left[f'(x_k^{*}) \right]^2} \Delta x_k = \int_a^b 2\pi f(x) \sqrt{1 + \left[f'(x) \right]^2} dx$$

revolving about x-axis

$$S = \int_a^b 2\pi f(x) \sqrt{1 + \left[f'(x) \right]^2} dx = \int_a^b 2\pi f(x) \sqrt{1 + \left(\frac{dy}{dx} \right)^2} dx$$

revolving about y-axis

$$S = \int_c^d 2\pi g(y) \sqrt{1 + \left[g'(y) \right]^2} dy = \int_c^d 2\pi g(y) \sqrt{1 + \left(\frac{dx}{dy} \right)^2} dy$$

Example 1

Find the area of the surface that is generated by revolving the portion of the curve $y = \sqrt{x}$ from $(1,1)$ *to* $(4,2)$ about *x-axis*

Example 2

Find the area of the surface that is generated by revolving the portion of the curve $x^2 = 16y$ from $(4,1)$ *to* $(12,9)$ about *y-axis*