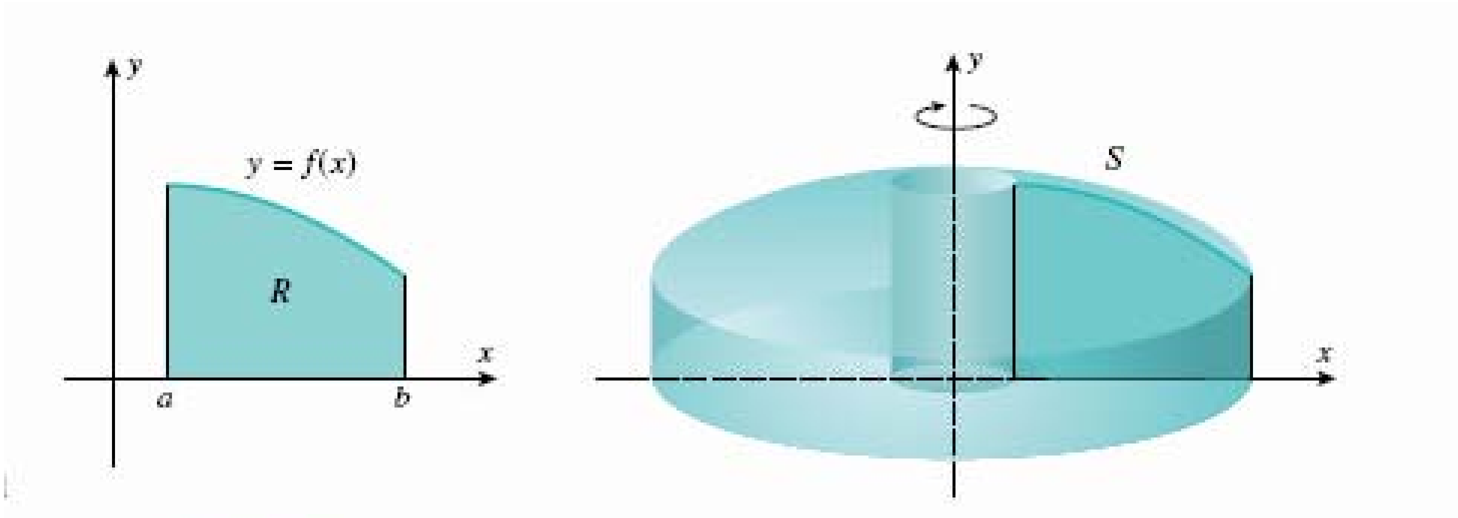
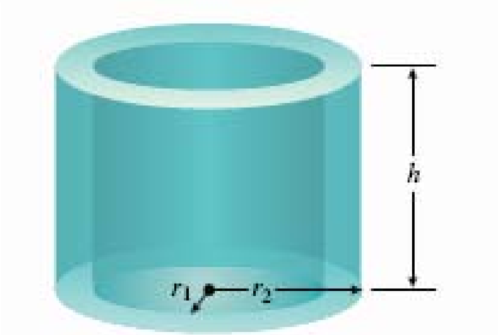
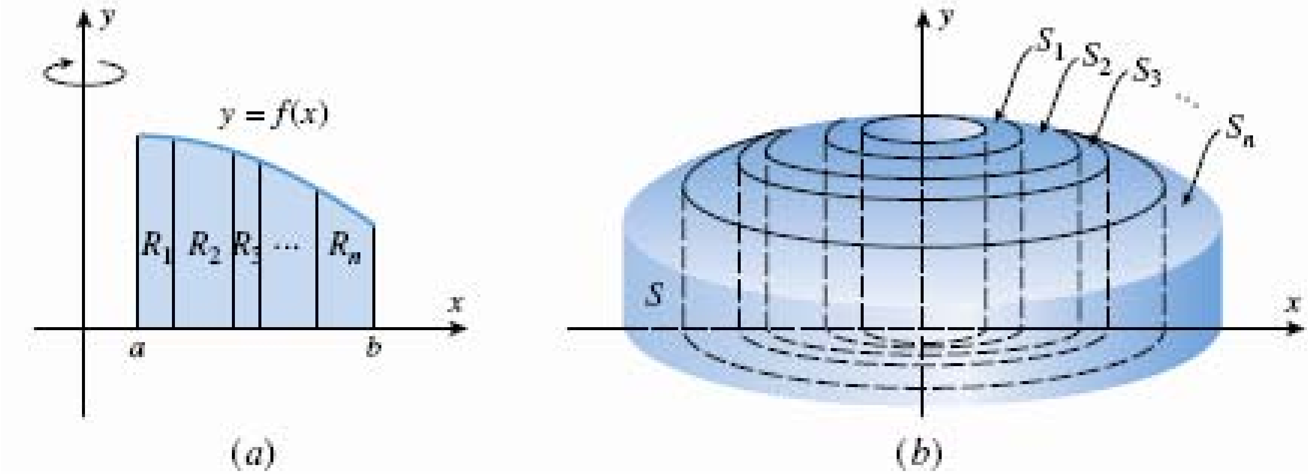
7.3 Volumes by Cylindrical Shells



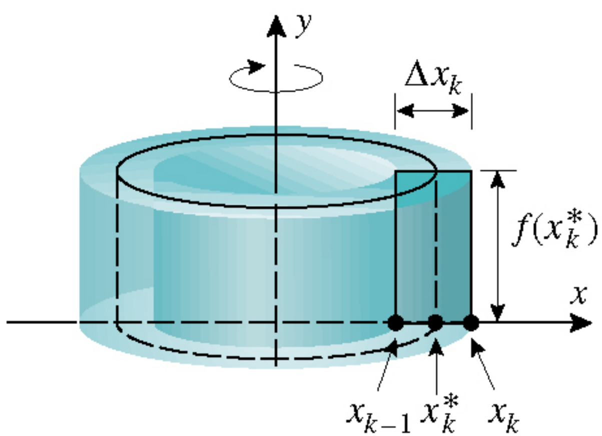


$$
\begin{aligned}
V &= [\text{area of cross section}][\text{height}] \\
&= \left(\pi r_2^2 - \pi r_1^2\right)h \\
&= \pi\left(r_2 + r_1\right)\left(r_2 - r_1\right)h \\
&= 2\pi\left[\frac{1}{2}\left(r_2 + r_1\right)\right]h\left(r_2 - r_1\right)
\end{aligned}
$$

$$V = 2\pi[\text{average radius}][\text{height}][\text{thickness}]$$



$$V = V(s_1) + V(s_2) + \cdots + V(s_n)$$



$$V_k = 2\pi x_k^* f(x_k^*)\Delta x_k$$

$$V \approx \sum_{k=1}^{n} 2\pi x_k^* f(x_k^*)\Delta x_k$$

$$V = \lim_{\max \Delta x_k \to 0} \sum_{k=1}^{n} 2\pi x_k^* f(x_k^*)\Delta x_k = \int_a^b 2\pi x f(x)\,dx$$

7.3.2 VOLUME BY CYLINDRICAL SHELLS ABOUT THE y-AXIS. Let f be continuous and nonnegative on $[a, b]$, and let R be the region that is bounded above by $y = f(x)$, below by the x-axis, and on the sides by the lines $x = a$ and $x = b$. Then the volume V of the solid of revolution that is generated by revolving the region R about the y-axis is given by

$$V = \int_a^b 2\pi x f(x)\, dx \tag{2}$$

Example 1

Use cylindrical shells to find the volume of the solid generated when the region enclosed between $y = \sqrt{x},\ x = 1,\ x = 4,\ and\ \ x - axis$ is revolved about the *y*-axis.

Example 2

Use cylindrical shells to find the volume of the solid generated when the region in first quadrant enclosed between $y = x,\ and\ \ y = x^2$ is revolved about the *y*-axis

Example 3

Use cylindrical shells to find the volume of the solid generated when the region under $y = x^2$, over the interval $[0, 2]$

is revolved about the line $y = -1$

“Only set up the integral”