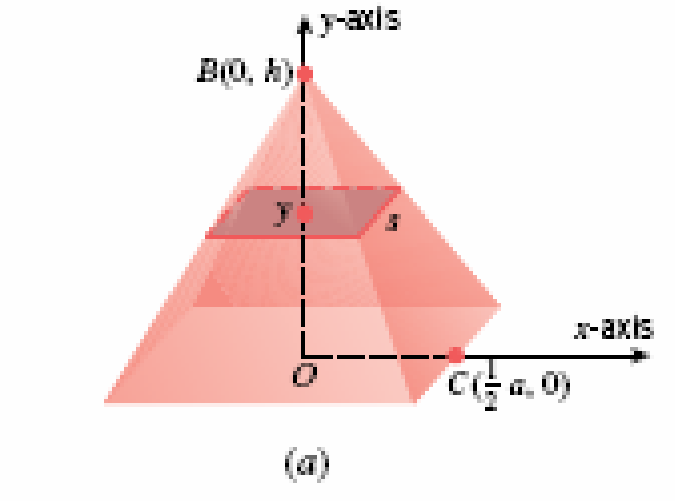
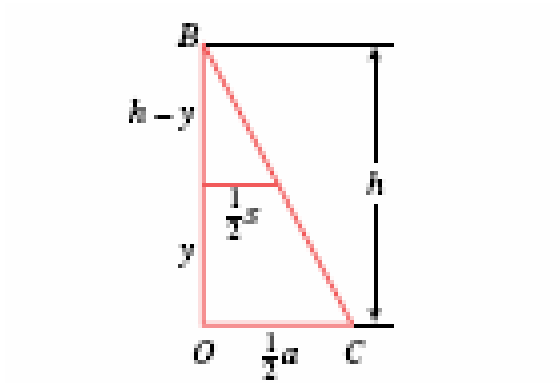
7.2 Volumes by Slicing; Disks and Washers

Example 1

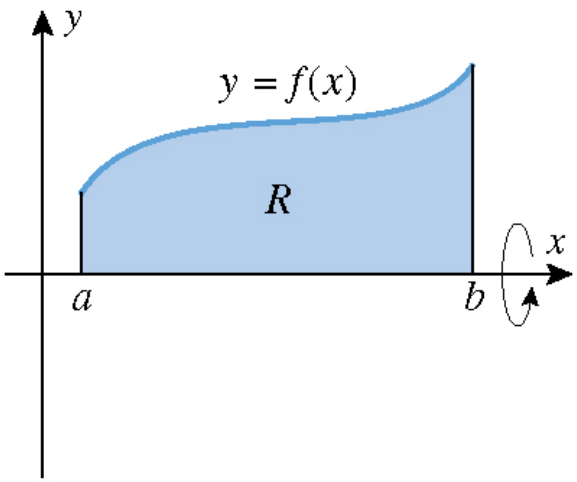
Derive the formula for the volume of a right pyramid whose altitude is *h* and whose base is a square with sides of length a.



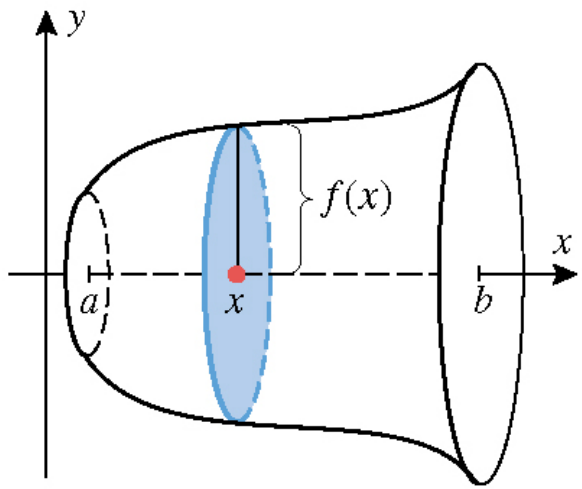


Volumes by Disks Perpendicular to the *x*-Axis

7.2.4 PROBLEM. Let f be continuous and nonnegative on $[a, b]$, and let R be the region that is bounded above by $y = f(x)$, below by the x-axis, and on the sides by the lines $x = a$ and $x = b$ (Figure 7.2.9*a*). Find the volume of the solid of revolution that is generated by revolving the region R about the x-axis.

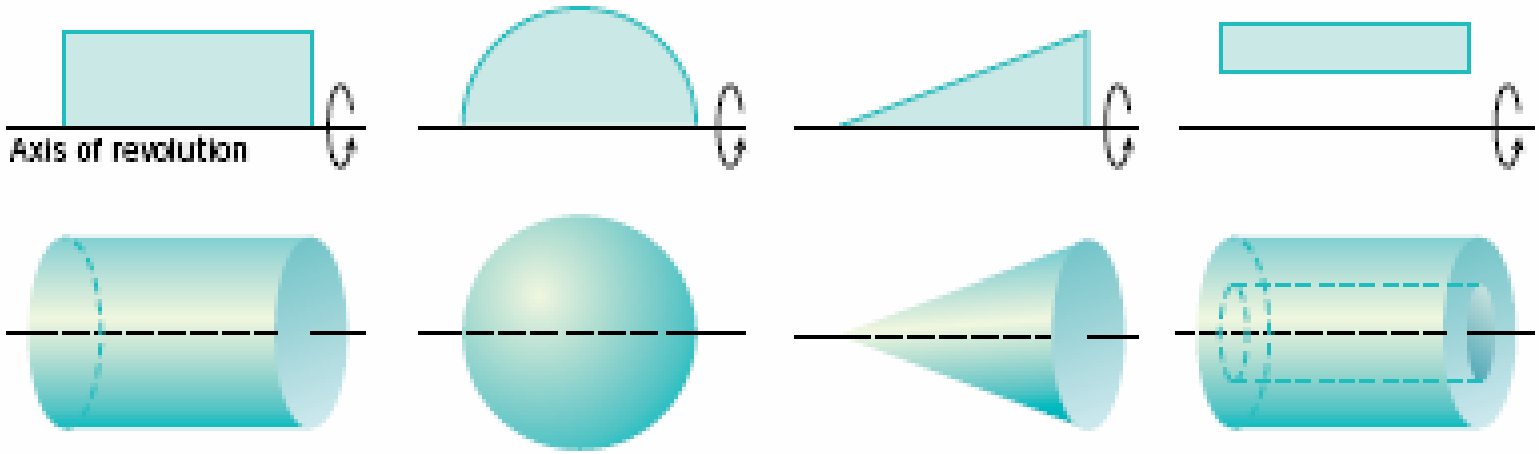


(*a*)



(*b*)

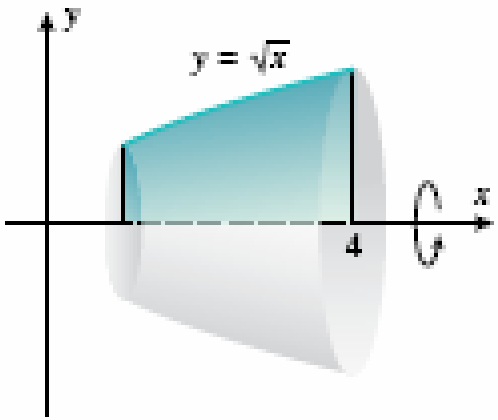
$$V = \int_a^b \pi [f(x)]^2 \, dx$$



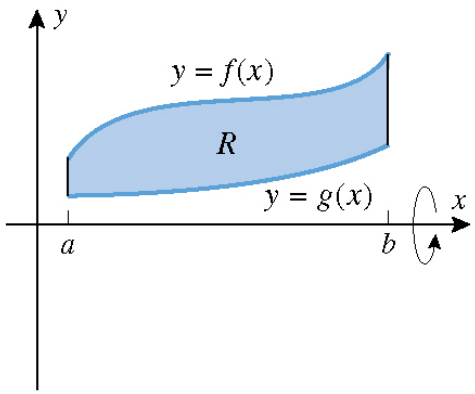
Some familiar solids of revolution

Example 2

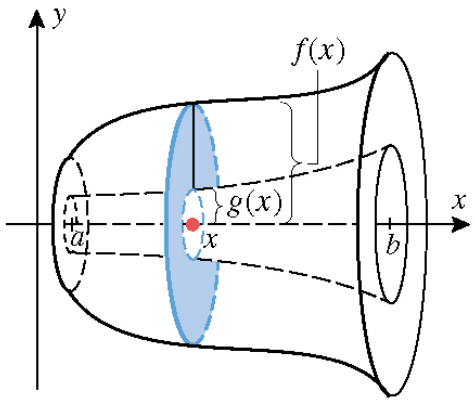
Find the volume of the solid that is obtained when the region under the curve $y = \sqrt{x}$ over the interval $[1,4]$ is revolved about *x*-axis.



Volumes by Washers Perpendicular to the *x*-Axis



(a)



(b)

$$V = \int_a^b \pi\left(\left[f(x)\right]^2 - \left[g(x)\right]^2\right) dx$$

Example 3

Find the volume of the solid generated when the region between the graphs $f(x) = \frac{1}{2} + x^2$ *and* $g(x) = x$ *over* $[0,2]$ is revolved about x-axis.

Volumes by Disks & Washers Perpendicular to the *y*-Axis

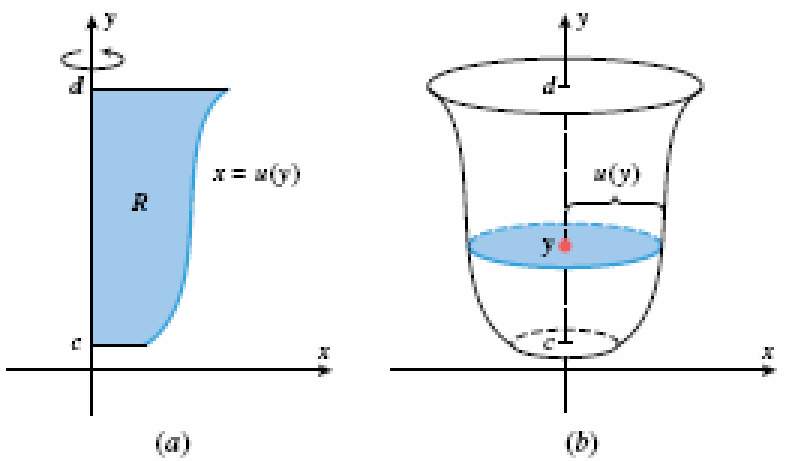


Figure 7.2.14

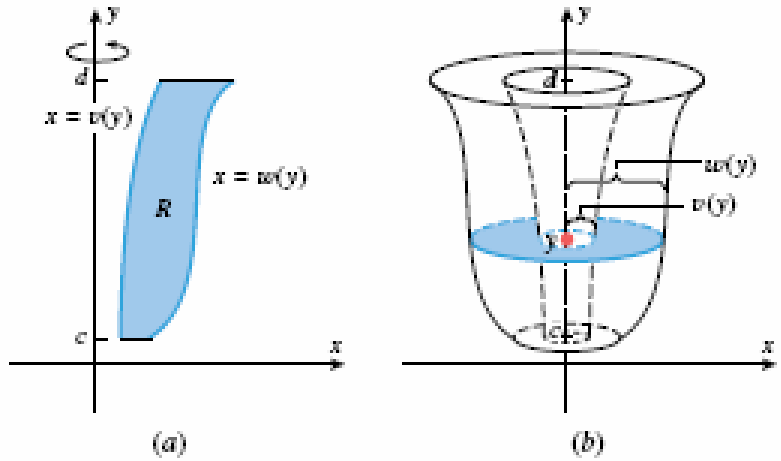


Figure 7.2.15

$$V = \int_c^d \pi[u(y)]^2\,dy$$

Disks

$$V = \int_c^d \pi([w(y)]^2 - [v(y)]^2)\,dy$$

Washers

Example 4

Find the volume of the solid generated when the region enclosed by $y = x^3$, $y = 1$, *and* $y = 8$ is revolved about the *y*-axis

Example 5

Find the volume of the solid generated when the region in the first quadrant enclosed by $y = x^3$, *and* $y = 4x$ is revolved about the line $x = 3$